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Multi-Criterion Dynamic User Equilibrium Models and Algorithms for Road Pricing Applications with Heterogeneous Users

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Why Road Pricing?

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- Motivating Phenomena:
  - Growing congestion in metropolitan areas…
  - Budget constraints for highway authorities…
- Objectives of road pricing:
  - Revenue generation: road/bridge tolls
  - Congestion management: congestion pricing, cordon tolls and high occupancy toll (HOT) lanes…toward dynamic pricing (with time-varying tolls)…
- Examples of road pricing applications
  - London cordon pricing: charging private vehicles in downtown area to reduce traffic congestion and raise revenues for transport improvements.
  - I-15 HOT lanes in San Diego: allowing solo drivers to pay a dynamic toll to use the express lanes normally reserved for high occupancy vehicles (HOV).
  - Highway 407, the Express Toll Route (ETR), in Toronto: collecting tolls based on distance traveled in the multi-lane electronic highway.
  - State Route 91 in Orange County, California: express toll lanes constructed and operated by private company.

Why is DUE Model important for Dynamic Pricing Applications?

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Estimated (dynamic) origin-destination (OD) demand

Network characteristics and traffic control data

Dynamic User Equilibrium Models
(or Dynamic Traffic Assignment)

- Address the dynamic nature of traffic flows
- Describe path choices of network users based on the user equilibrium/optimal
- Obtain DUE path flow patterns

Time-varying road pricing scenario (as a set of time-dependent link tolls)

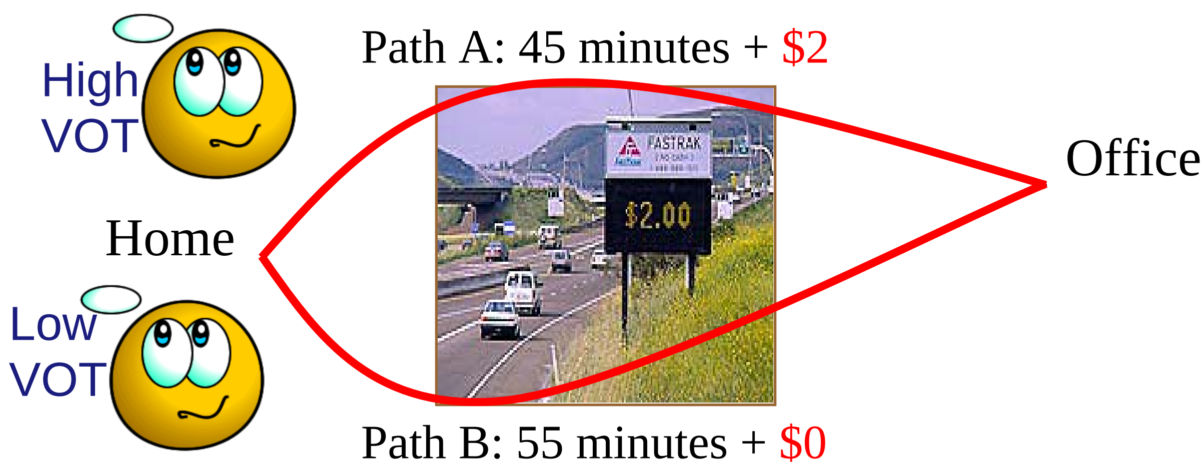
Evaluation of dynamic pricing scenarios: predicted path choices (toll road usage, profit and revenue) and network performance (average travel time and average delay).

Decision support for toll operators and traffic system managers

Which Essential Aspect Was Not Considered?

User Heterogeneity

- Critical limitation of existing dynamic traffic assignment tools
  - Each trip-maker chooses a path that minimizes the two major path travel criteria: travel time and out-of-pocket cost (path generalized cost).
  - Conventional traffic assignment models consider a homogeneous perception of tolls by assuming a constant VOT in the path choice model.
  - Empirical studies (e.g. Hensher, 2001; Brownstone and Small 2005; Cirillo et al. 2006) found that the VOT varies significantly across individuals.



Our Objective

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- Develop Multi-Criterion Simultaneous Route and Departure Time User Equilibrium (MSRDUE) models and algorithms
  - Address the heterogeneous user preference of path and/or departure time choices in response to time-varying toll charges.
  - Capture traffic flow dynamics and spatial and temporal vehicular interactions (simulation-based approach).
  - Adhere to the time-dependent generalization of Wardrop's UE principle (gap function measures the deviation from equilibrium).
  - Be deployable on road traffic networks of practical sizes (vehicle-based implementation technique).

Problem Statement

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- Assumptions:
  - G(N, A), discretized planning horizon, and time-dependent link tolls.
  - Define schedule delay as the difference between actual and preferred arrival times (PAT).
    - Every trip-maker has his/her own PAT interval θ
    - Early schedule delay (ESD) and late schedule delay (LSD)
    - Value of ESD (VOESD β) and value of LSD (VOLSD λ)
  - The experienced trip cost perceived by a trip-maker with θ, α, β, and λ

$$G_{odp}^{\tau}(\theta,\alpha,\beta,\lambda) = \underbrace{TC_{odp}^{\tau} + \alpha \times TT_{odp}^{\tau}}_{\text{Path generalized cost}} + \underbrace{\beta \times ESD_{odp}^{\tau}(\theta) + \lambda \times LSD_{odp}^{\tau}(\theta)}_{\text{Schedule delay cost}}$$

where $ESD_{odp}^{\tau}(\theta) = \max\{0, \theta^{lb} - \tau^{mid}\}$ $LSD_{odp}^{\tau}(\theta) = \max\{0, \tau^{mid} - \theta^{ub}\}$

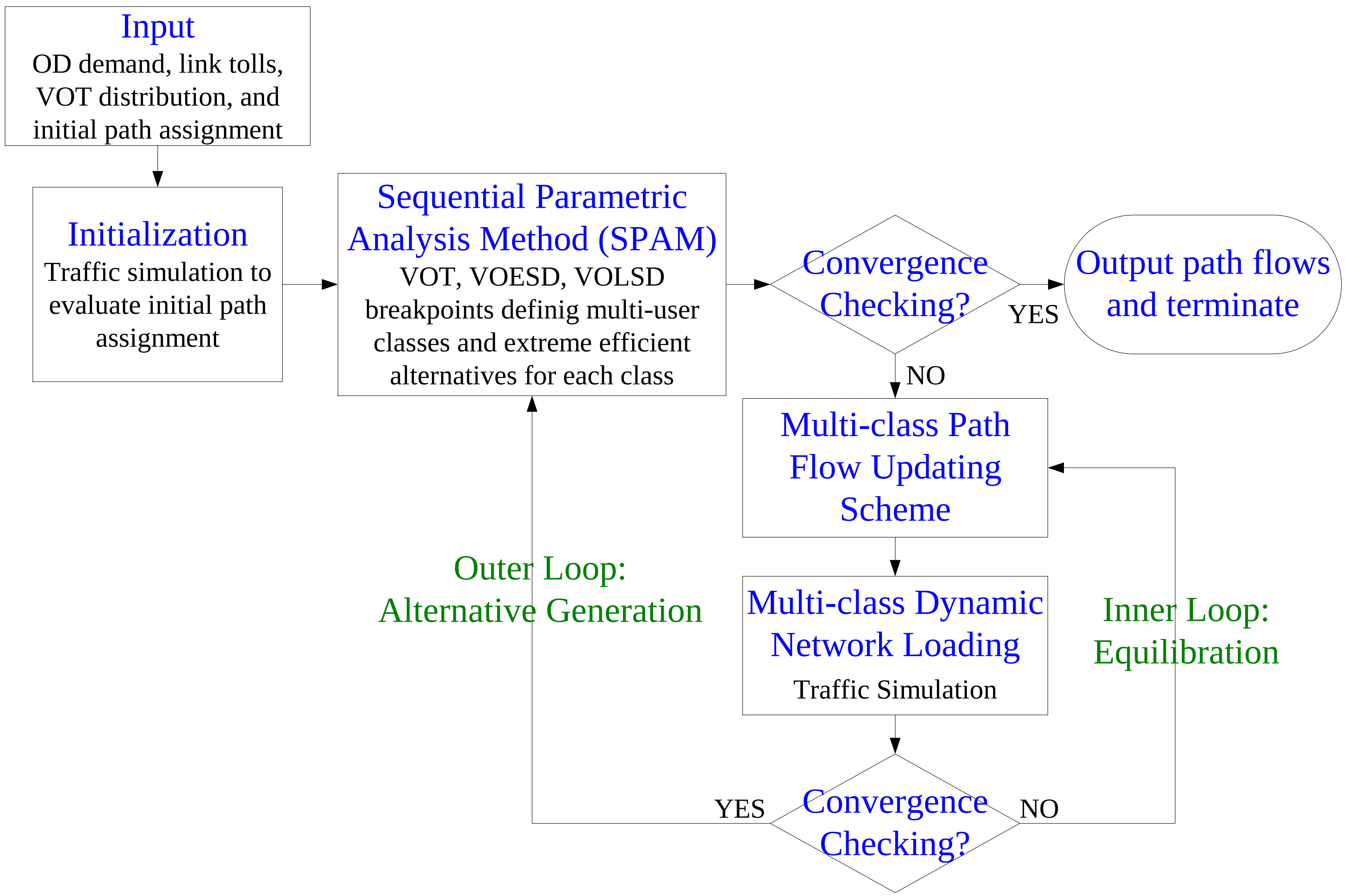
  - VOT α, VOESD β, and VOLSD λ are continuously distributed across trip-makers with given probability density functions and feasible ranges.

Problem Statement (ctd.)

- Departure time and path choice behavioral assumption:
  - Each trip-maker chooses the alternative that minimizes the experienced trip cost with respect to his/her PAT, VOT, VOESD, and VOLSD.
  - An alternative is a combination of arrival time interval and the corresponding least generalized cost path (that arrives the destination at that arrival time interval).
- Multi-criterion simultaneous route and departure time UE (MSRDUE)
  - For each OD pair, every trip cannot decrease the experienced trip cost with respect to that trip's particular VOT, VOESD, VOLSD, and PAT interval by unilaterally changing departure time and/or path.
    - Each trip-maker is assigned to the alternative that has the least trip cost with respect to his/her own PAT, VOT, VOESD, and VOLSD.
- MSRDUE problem:

  Under a given time-dependent road pricing scenario, to solve for the departure time and path flow patterns satisfying the MSRDUE conditions.

Column Generation-based MSRDUE algorithm



Sequential Parametric Analysis Method (SPAM)

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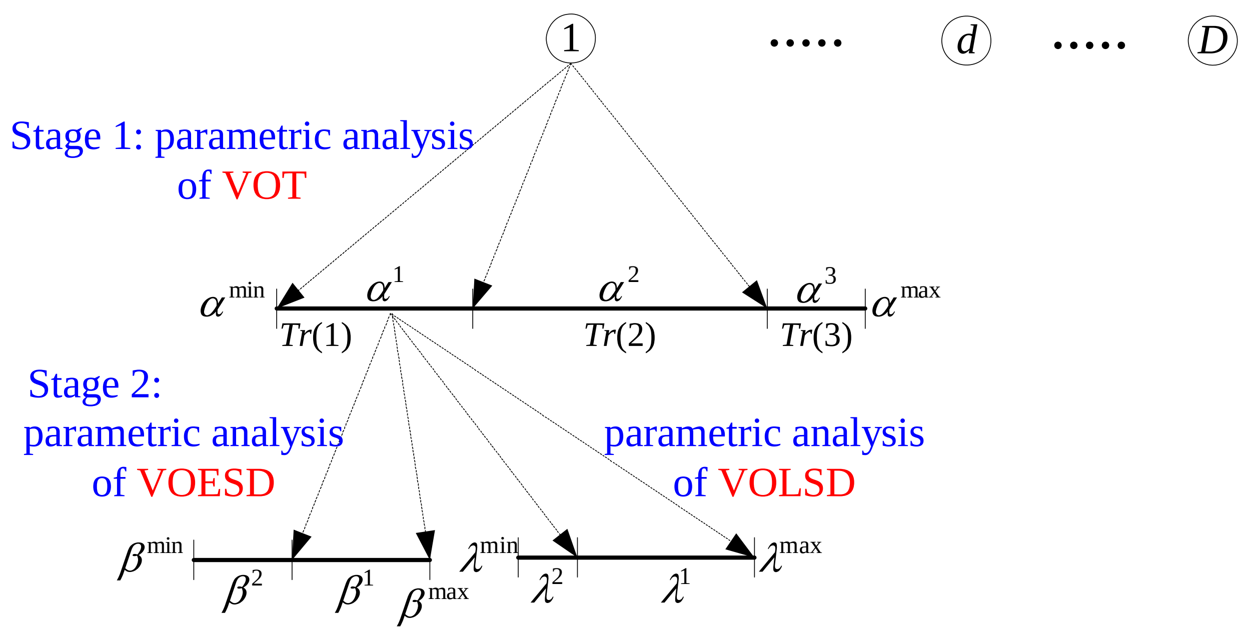
- Determine VOT, VOESD, and VOLSD breakpoints that define multi-user classes, and find the least trip cost (extreme non-dominated) alternative for each user class

Repeat the two stages for each destination: d = 1,...,D



Repeat the second stage for each VOT subinterval: b=1,...,3

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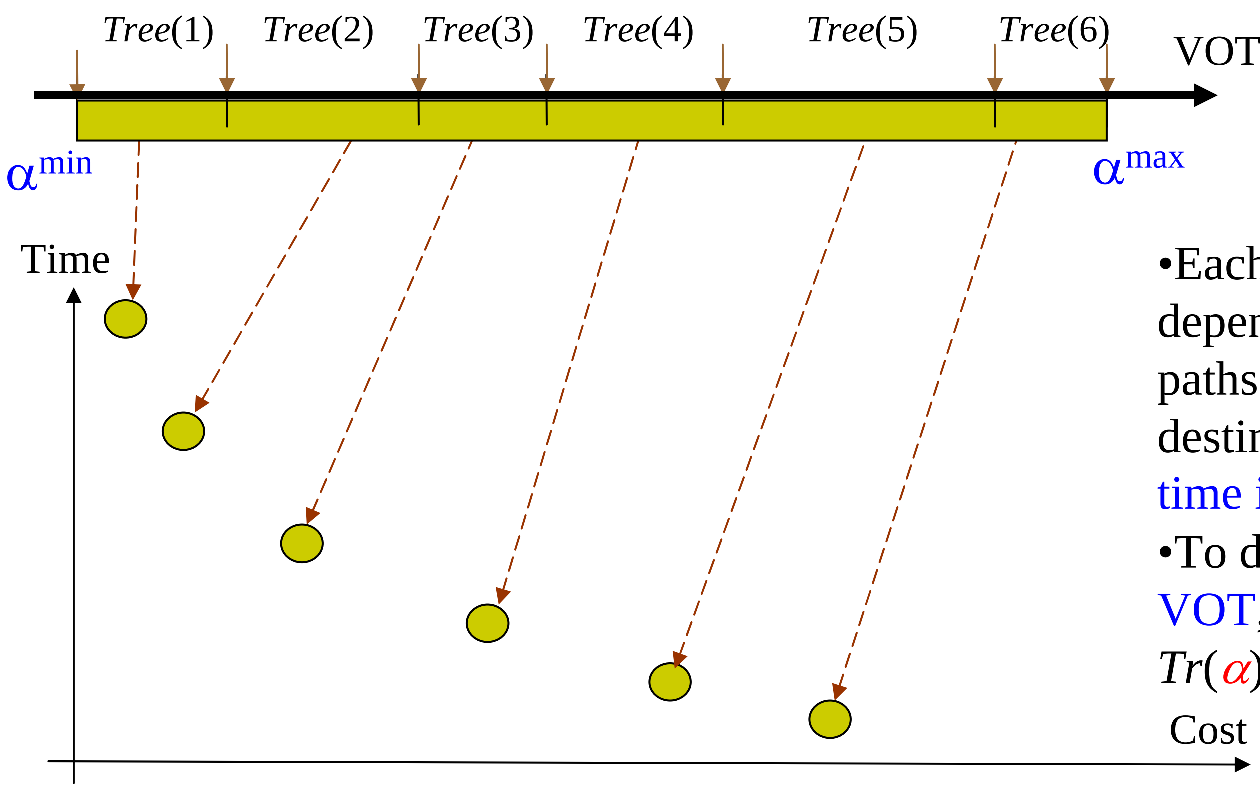
Determine the breakpoints that partition the feasible VOT range and define the master user classes, and find time-dependent least generalized cost path tree for each user class.



- Each tree consists of time-dependent least generalized cost paths from all origin nodes to a destination node, for all arrival time intervals.
- To determine the subinterval of VOT, in which the current tree $Tr(\alpha)$ is optimal.

Parametric Analysis of VOESD and VOLSD for a VOT subinterval – stage 2 of the SPAM

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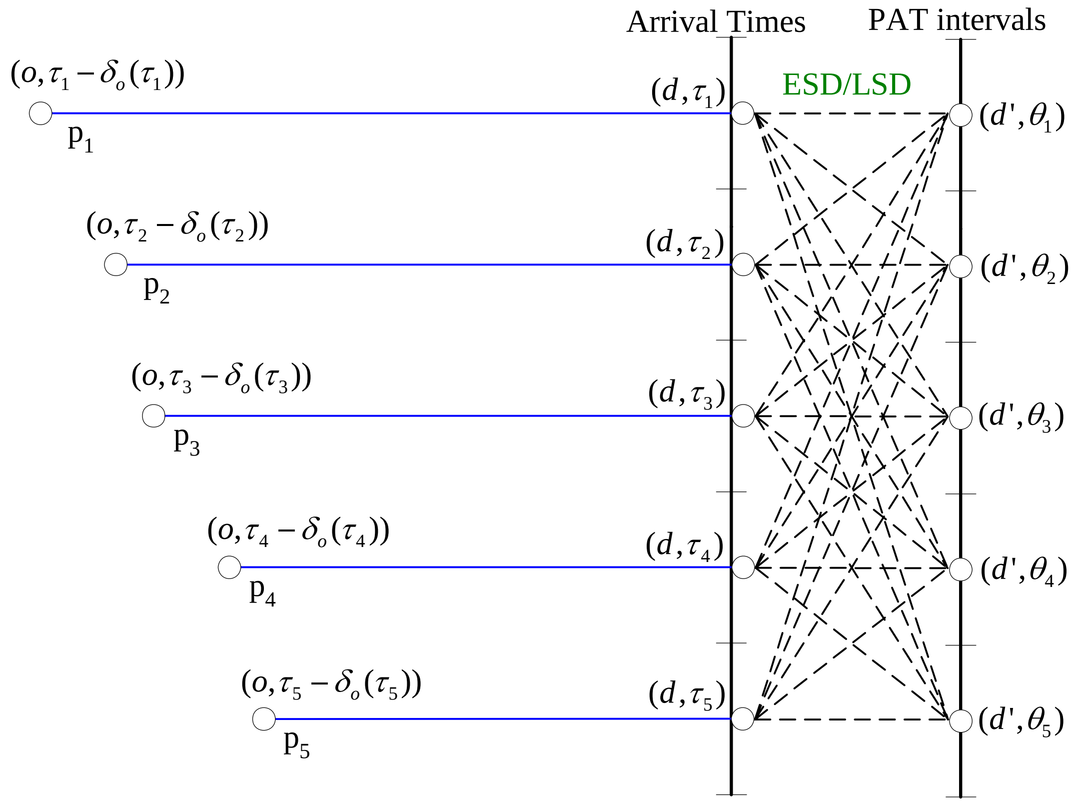
- Given a time-dependent extreme efficient path tree *Tr*(*b*) corresponding to the VOT subinterval $[\alpha^{b-1}, \alpha^{b})$, the parametric analyses of VOESD and VOLSD are conducted in an expanded network.



Parametric Analysis of VOESD and VOLSD for a VOT subinterval

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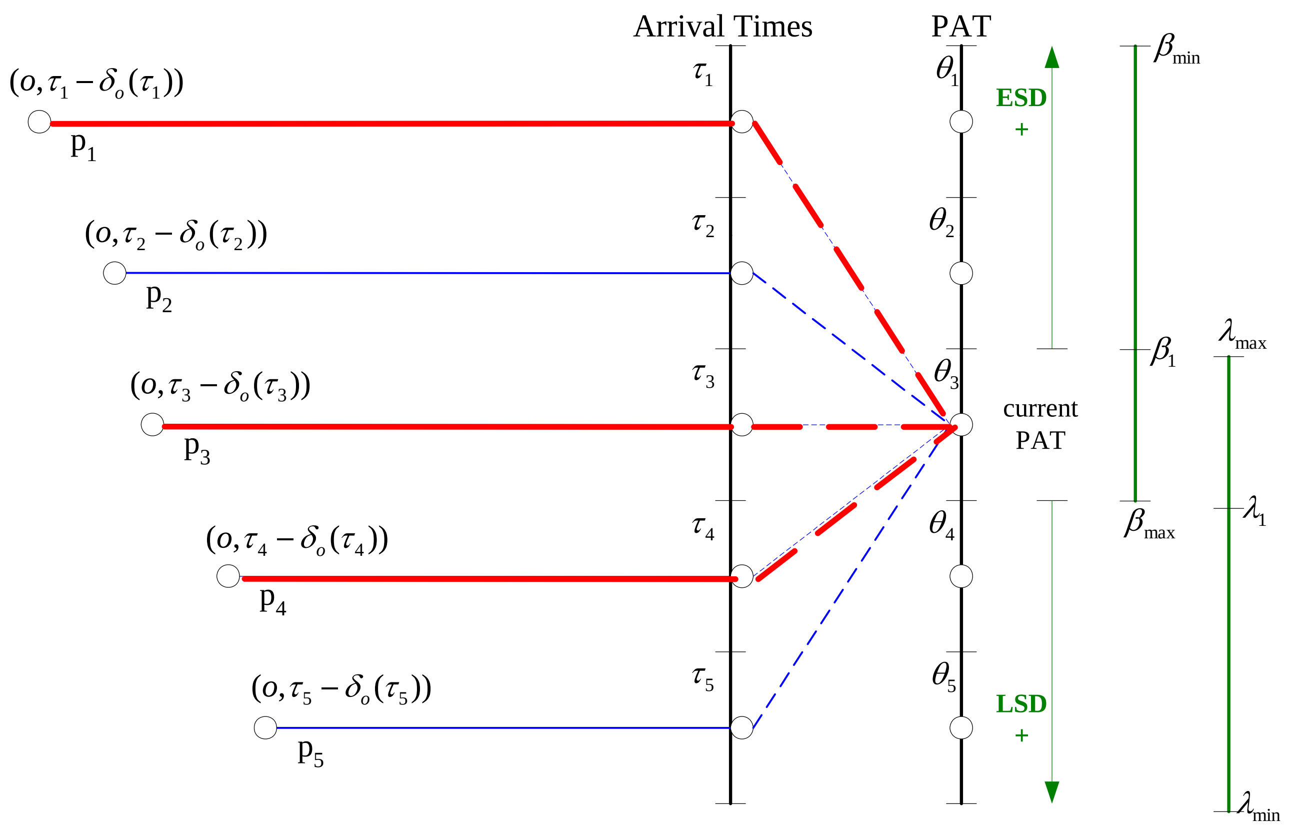
- An example



Parametric Analysis of VOESD and VOLSD for a VOT subinterval

- Output of the SPAM
  - VOESD breakpoints that define the subintervals, and the least trip cost alternative for each subinterval. $\forall b, \forall \theta,$

$$\beta(b,\theta) = \{\beta^0, \beta^1, ..., \beta^{M(b,\theta)} \mid \beta^{\max} = \beta^0 > \beta^1 > ... > \beta^m > ... > \beta^{M(b,\theta)} = \beta^{\min}\}$$

$$[\beta^{m-1}, \beta^m)_{b,\theta}, \ (\tau^*, p^*)_{b,\theta,m}, \quad m = 1, ..., M(b,\theta)$$

  - VOLSD breakpoints that define the subintervals, and the least trip cost alternative for each subinterval. $\forall b, \forall \theta,$

$$\lambda(b,\theta) = \{\lambda^0, \lambda^1, ..., \lambda^{N(b,\theta)} \mid \lambda^{\max} = \lambda^0 > \lambda^1 > ... > \lambda^n > ... > \lambda^{N(b,\theta)} = \lambda^{\min}\}$$

$$[\lambda^{n-1}, \lambda^n)_{b,\theta}, \ (\tau^*, p^*)_{b,\theta,n}, \quad n = 1, ..., N(b,\theta)$$

- Multiple user classes: for each VOT subinterval b and PAT θ,

$$u(b,\theta, m_{\beta(b,\theta)}, n_{\lambda(b,\theta)}), \ m = 1, ..., M(b,\theta), \ n = 1, ..., N(b,\theta)$$

  - Simplified as $u(b, \theta, m, n)$
  - The corresponding set of least trip cost alternatives

$$alt_{od}(b,\theta,m,n) = alt_{od}(b,\theta,m_{b,\theta}) \cup alt_{od}(b,\theta,n_{b,\theta})$$

Multi-Class Flow Updating and Convergence Checking

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- Multi-Class Alternative Flow Updating Scheme
  - Multiple user classes $u(b, \theta, m, n)$ are naturally determined by the SPAM.
  - Decomposes the problem into many (b,θ,m,n,o,d) sub-problems and solves each of them by adjusting OD flows between non-least trip cost alternatives and the least trip cost alternative.
  - Extension of the multi-class path flow updating scheme for the BDUE
- Convergence Checking
  - Gap

$$Gap(r^l) = \sum_{u(b,\theta,m,n)} \sum_{o} \sum_{d} \sum_{(\tau,p)\in alt_{od}(b,\theta,m,n)} r_{odp}^{\tau,l}(b,\theta,m,n) \times \Delta_{odp}^{\tau,l}(b,\theta,m,n)$$

  - Average Gap

$$AGap(r) = \frac{\sum_{u(b,\theta,m,n)} \sum_{o} \sum_{d} \sum_{(\tau,p)\in alt_{od}(b,\theta,m,n)} r_{odp}^{\tau,l}(b,\theta,m,n) \times \Delta_{odp}^{\tau,l}(b,\theta,m,n)}{\sum_{u(b,\theta,m,n)} \sum_{o} \sum_{d} \sum_{(\tau,p)\in alt_{od}(b,\theta,m,n)} r_{odp}^{\tau,l}(b,\theta,m,n)}$$

Numerical Experiments and Results

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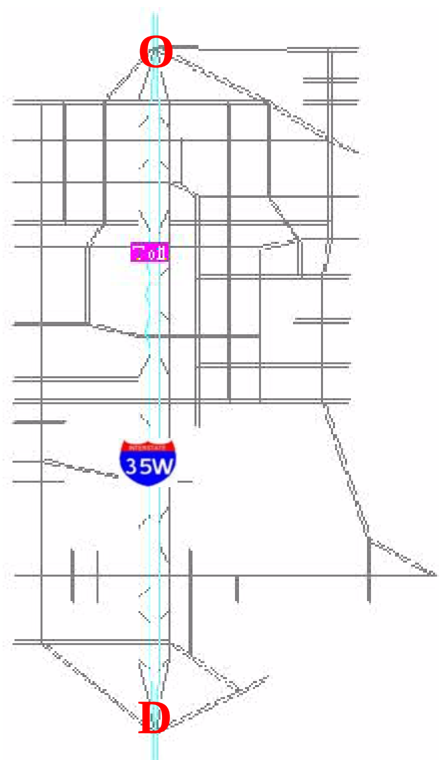
- Purpose
  - Examine the algorithmic convergence property and solution quality of the algorithm
  - Investigate how the random parameters would affect departure time and path flow patterns (or toll road usage) under different dynamic pricing scenarios (i.e. to compare the random and constant parameter models).
- Random parameters
  - VOT distribution: N($0.4/min, $0.2/min), $[\alpha^{\min}, \alpha^{\max}] = [0.01, 3.0]$

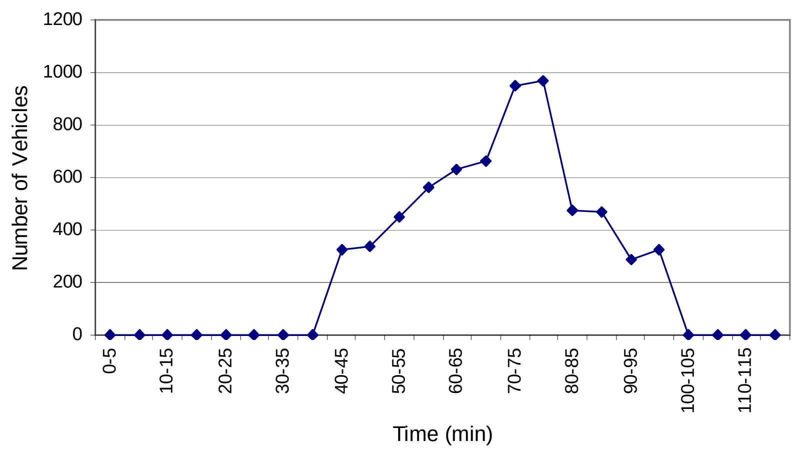
    (Lam and Small, 2001; Brownstone and Small, 2005; Southern CA)
  - VOESD distribution: N($0.3/min, $0.15/min), $[\beta^{\min}, \beta^{\max}] = [0.01, 2.0]$
  - VOLSD distribution: N($1.8/min, $0.6/min), $[\lambda^{\min}, \lambda^{\max}] = [0.25, 4.0]$

    (economic judgments based on the results reported in Small (1982))
- Arrival time and PAT intervals: 5 minutes.

Numerical Experiments and Results

- Experiment conducted on the Fort Worth network (TX)
  - Select a critical OD pair that accounts for 25% of total demand.





PAT pattern

Pricing Scenario	0-20 minutes	20-40 minutes	40-60 minutes	60-80 minutes	80-100 minutes	100-120 minutes	120-150 minutes
#1 (low)	$0.05	$0.20	$0.35	$0.50	$0.35	$0.20	$0.05
#2 (mid)	$0.25	$0.40	$0.55	$0.70	$0.55	$0.40	$0.25
#3 (high)	$0.45	$0.60	$0.75	$0.90	$0.75	$0.60	$0.45

dynamic pricing scenarios

Numerical Experiments and Results

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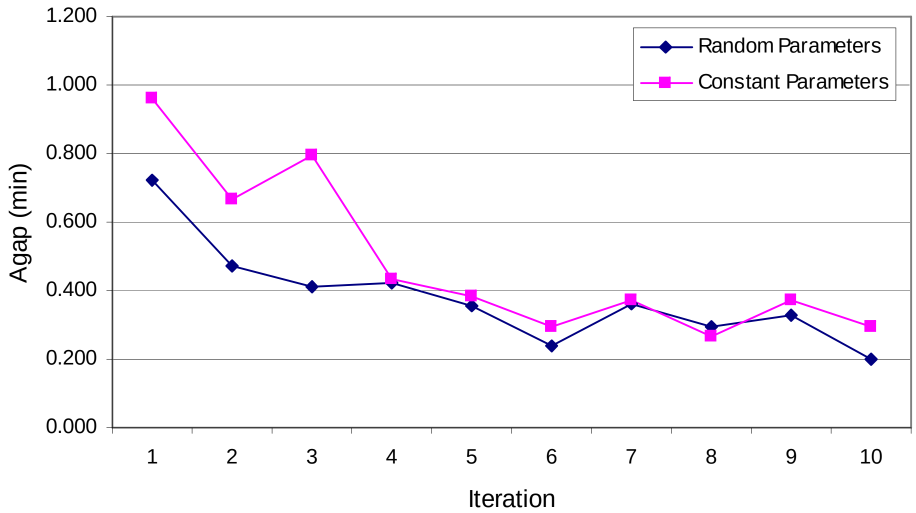
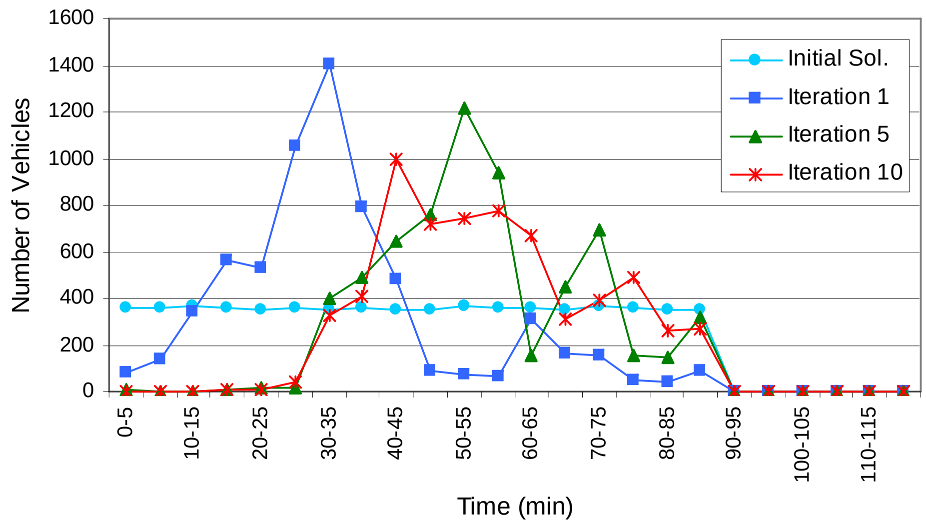
- Experiment conducted on the Fort Worth network (TX)
  - Convergence pattern and solution quality in terms of Average Gap.
  - Convergence pattern in terms of departure time distribution



average gap



departure time distribution (random parameter model)

Numerical Experiments and Results

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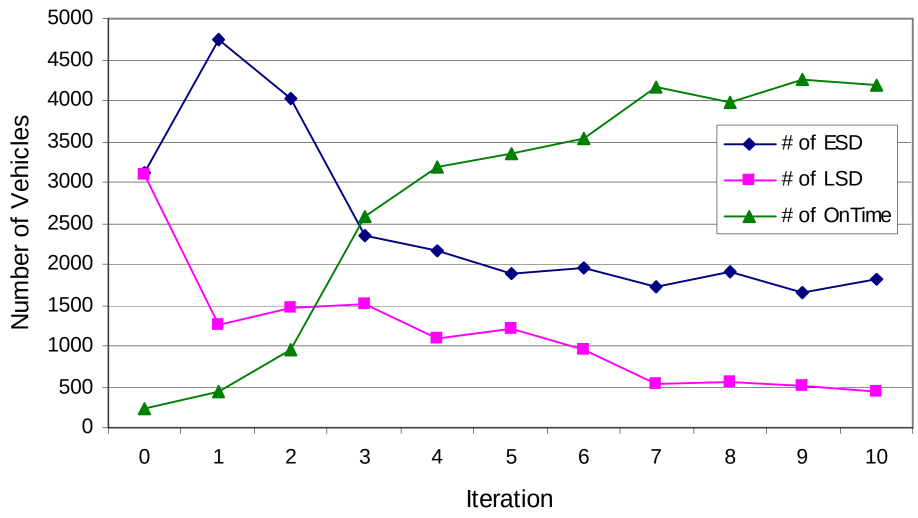
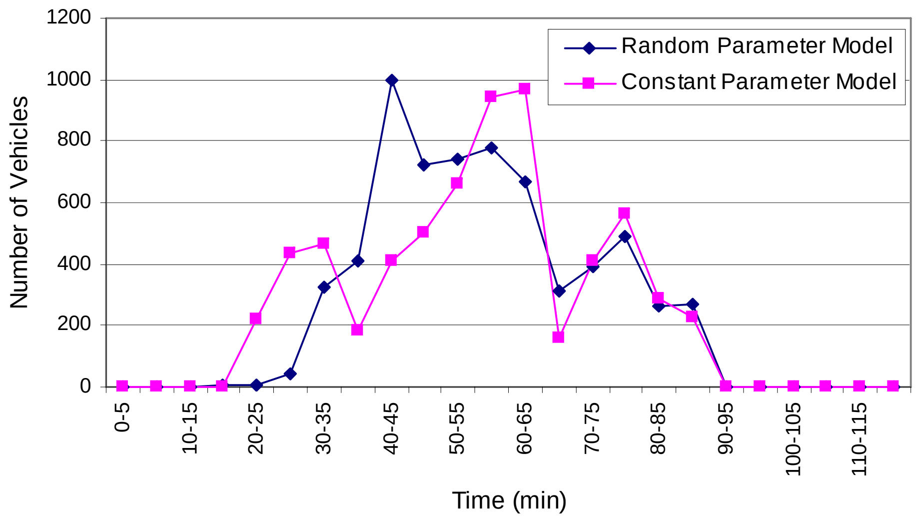
- Experiment conducted on the Fort Worth network (TX)
  - Convergence pattern in terms of the number of schedule delay vehicles (i.e. early, late, and on-time vehicles) in the random parameter model

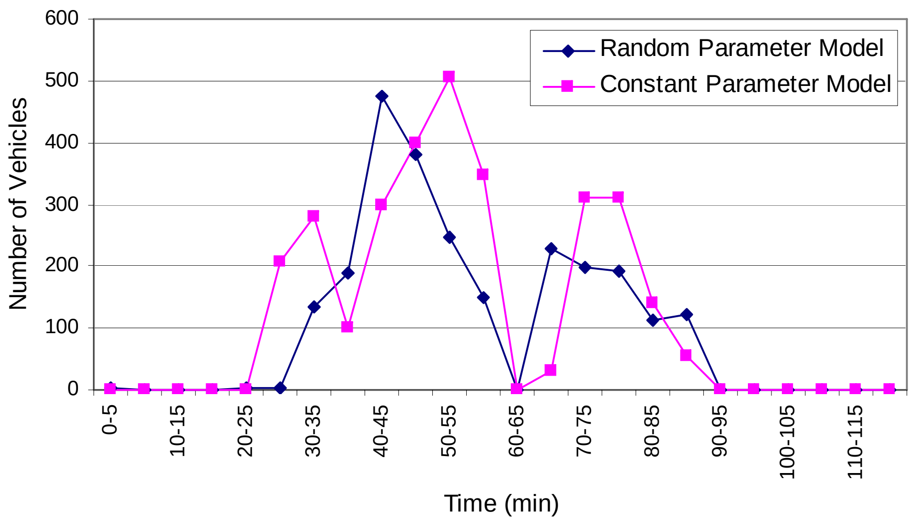


Numerical Experiments and Results

- Experiment conducted on the Fort Worth network (TX)
  - Compare the differences in departure time distribution and toll road usage between random and constant parameter models



departure time distribution



Time-varying toll road usage

Numerical Experiments and Results

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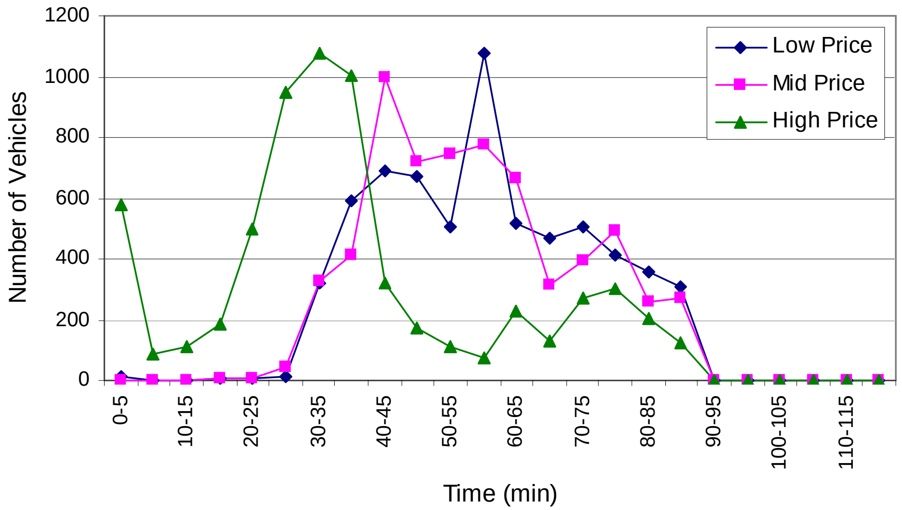
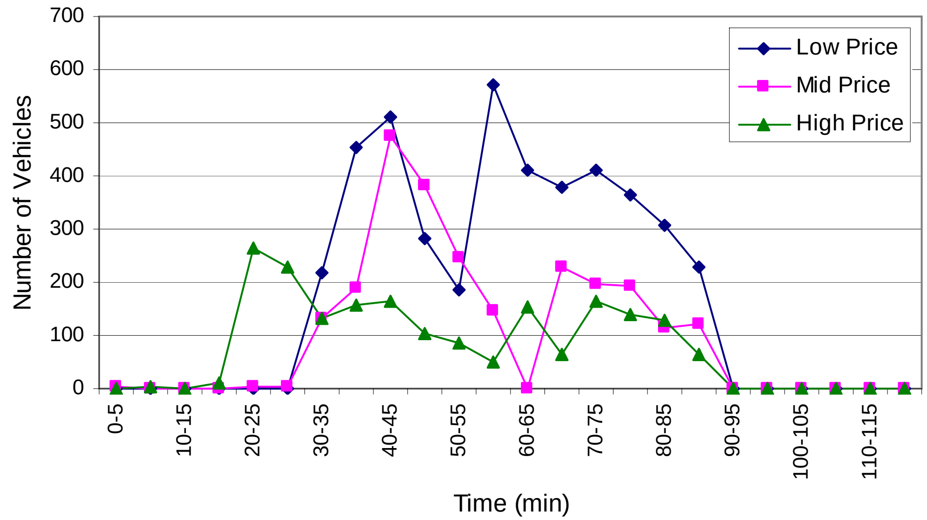
- Experiment conducted on the Fort Worth network (TX)
  - The Comparison of departure time distribution and toll road usage under different dynamic pricing scenarios



departure time distribution



Time-varying toll road usage

Concluding Remarks

- VOT and VOSD are assumed continuously distributed across trip-makers.
- The MSRDUE problem is solved by the column generation-based algorithm which embeds the extreme non-dominated path finding algorithm – SPAM (sequential parametric analysis method), in addition to the multi-class alternative flow updating scheme and the traffic simulator
- The algorithm is independent of the VOT, VOESD, and VOLSD assumptions, and independent of the traffic simulator.
    - The convergence pattern of the proposed MSRDUE algorithm is not affected by the different assumptions of VOT, VOESD, and VOLSD, and it is able to find close-to-MSRDUE solutions.

Concluding Remarks

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- There are significant differences in the estimated/predicted departure time pattern and toll road usage between the two models.
  - Trip-makers behave identically in choosing departure times and paths in the constant parameter model.
  - The random parameter model explicitly considers heterogeneous users with different parameters.
- The proposed MSRDUE model can realistically describe trip-makers' responses to time-varying toll charges in temporal distribution (departure times) and spatial splits (path flows).
- Future Works:
  - Inclusion of other path choice attributes, such as reliability.
  - Extensions to OD-specific and time-varying VOT, VOESD, and VOLSD distributions
  - Development of re-optimization algorithms for the PAM and SPAM
  - Applications to dynamic congestion pricing problems


THE
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