Gap-driven Algorithms for the Time-Dependent User Equilibrium Problem on Large Networks

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Motivation and Objective (1/3)

- Time-dependent user equilibrium (TDUE) traffic assignment
  - Also called dynamic user equilibrium (DUE; Smith, 1995)
  - Address the dynamic nature of traffic flows as well as the path and/or departure time choices of network users.
  - Wide practical use for predicting dynamic traffic flow patterns in evaluating traffic control and demand management measures.
- Modeling and solution approaches
  - Analytical
  - Simulation-based

Motivation and Objective (2/3)

- TDUE models for dynamic road pricing applications should essentially be able to
  - Capture traffic flow dynamics and spatial and temporal vehicular interactions (simulation-based TDUE approach).
  - Adhere to the time-dependent generalization of Wardrop's UE principle (gap function measures the deviation from equilibrium).
  - Be deployable on road traffic networks of practical sizes (vehicle-based implementation technique).

Motivation and Objective (3/3)

- Improve theoretical basis of the simulation-based approach
  - Analytical TDUE approach
    - Well-defined link/node exit constraints and travel time functions; existence and uniqueness; adherence to the TDUE conditions.
    - Theoretical elegance is obtained at the cost of losing traffic realism.
  - Simulation-based TDUE approach
    - Describe traffic flow propagation, capture vehicular interactions and determine travel times through traffic simulations.
    - Heuristics do not guarantee convergence to TDUE solutions.
- Objective: theoretically-sound simulation-based TDUE approach
  - Capture traffic flow dynamics in simulation

Assumptions, Definition, & Problem Statement

- Assumptions
  - $G(N, A)$ and discretized horizon - time-varying link travel time/cost $c_{ij}(t)$
  - Time-dependent OD demands are known (i.e., fixed departure times).
  - Individual trip-makers minimizes his/her experienced path travel times.
- Time-Dependent User Equilibrium (TDUE)
  - For each OD pair and each departure time interval, no traveler can reduce his/her experienced path time by unilaterally changing path.
  - All Trips are assigned to the least experienced time paths.
- Problem Statement
  - Obtain a time-varying path flow pattern satisfying the TDUE conditions

Gap-based TDUE Formulation

- Define the gap function

$$Gap(r) = \sum_{o\in O}\sum_{d\in D}\sum_{\tau\in T}\sum_{p\in P(o,d,\tau)} r_{odp}^{\tau}[c_{odp}^{\tau}(r) - \pi_{od}^{\tau}(r)]$$

  - A measure of the violation of the TDUE conditions for a given time-varying path flow pattern $r \in \Omega$.
  - Solving the DUE problem is equivalent to finding the path flow vector $r^* \in \Omega$ such that $Gap(r^*) = 0$.
- Nonlinear Minimization Problem (NMP)

$$\underset{r\in\Omega}{Min} \sum_{o\in O}\sum_{d\in D}\sum_{\tau\in T}\sum_{p\in P(o,d,\tau)} r_{odp}^{\tau}[c_{odp}^{\tau}(r) - \pi_{od}^{\tau}(r)]$$

Subject to $\sum_{p\in P(o,d,\tau)} r_{odp}^{\tau} = q_{od}^{\tau} \quad \forall o,d,\tau$

$$c_{odp}^{\tau}(r^*) - \pi_{od}^{\tau}(r^*) \ge 0 \quad r_{odp}^{\tau} \ge 0 \quad \forall o,d,\tau, p\in P(o,d,\tau)$$

TDUE Solution Algorithm (1/4)

- Column generation-based algorithmic framework
  - NMP is path flow-based; difficult to enumerate all paths in large networks.
    - Generate a representative subset of paths.
  - Augments, in the outer loop, the subset of feasible paths and solves, in the inner loop, the restricted NMP (RNMP) with the current subset of paths.
    - A time-dependent shortest path algorithm (Ziliaskopoulos and Mahmassani, 1993) to find paths with competitive travel times.
    - A traffic simulator – DYANSMART (Jayakrishnan et al. 1994; Mahmassani, 2001) to determine the experienced path times $c(r)$.
    - A route swapping-based descent direction method to solve the RNMP.

TDUE Conditions

- TDUE conditions
  - The time-varying path flow vector $r^* \in \Omega$ is a solution to the TDUE problem if the following TDUE conditions are satisfied:

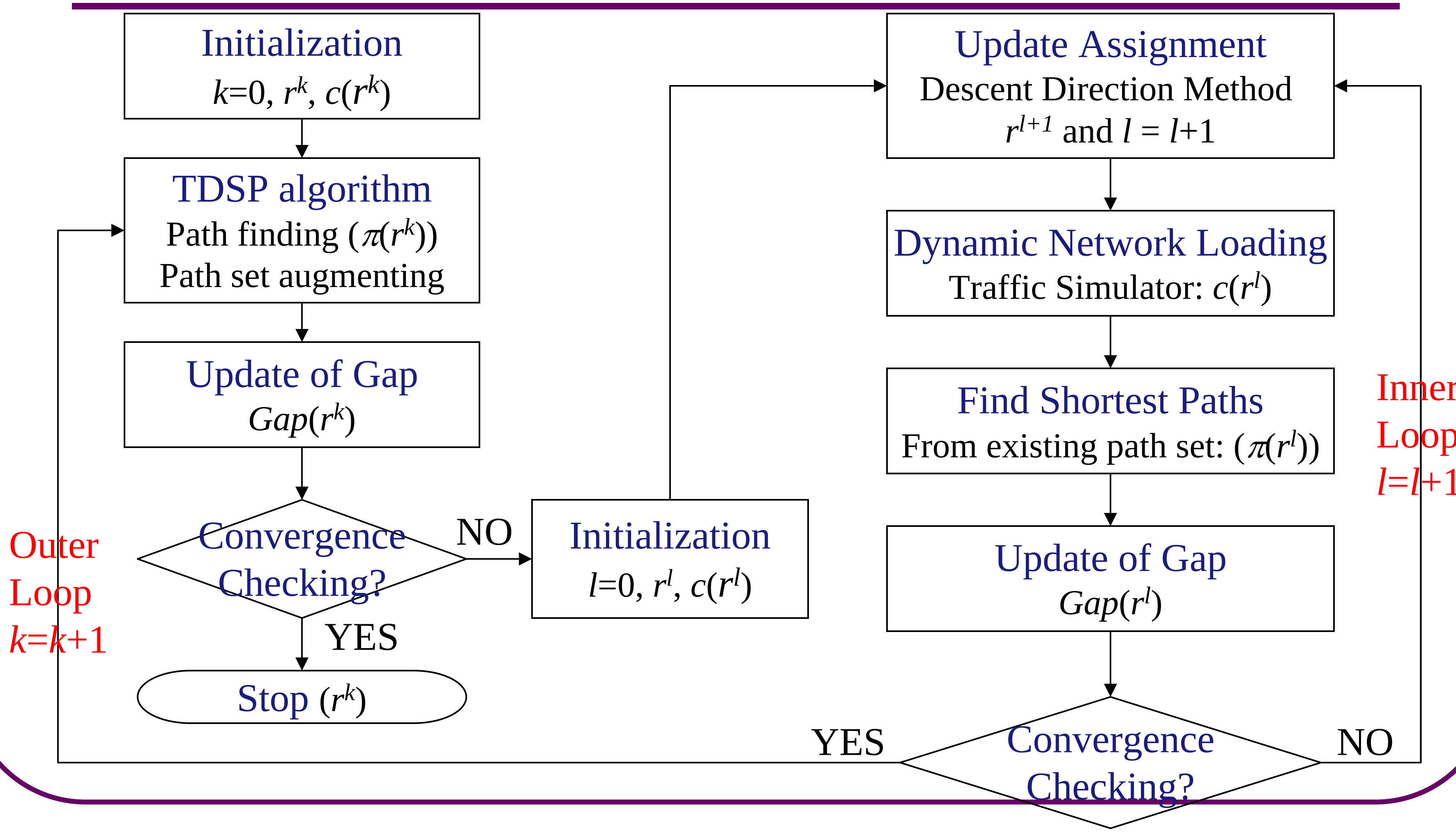
$$r_{odp}^{\tau *} \times [c_{odp}^{\tau}(r^*) - \pi_{od}^{\tau}(r^*)] = 0 \qquad \text{Complementary Slackness}$$

$$c_{odp}^{\tau}(r^*) - \pi_{od}^{\tau}(r^*) \geq 0 \qquad \forall o, d, \tau \text{ and } p \in P(o, d, \tau)$$

  - r_{odp}^{τ} is the number of vehicles from *o*, at time τ, to *d* on path *p*,
  - $c_{odp}^{\tau}(r)$ is the path travel time from *o*, at time τ, to *d* on path *p*, w.r.t. *r*,
  - $\pi_{od}^{\tau}(r)$ is the least travel time from *o*, at time τ, to *d*, w.r.t *r*.
  - $\Omega \equiv \{r\}$, is the set of feasible path flow vectors, satisfying the path flow conservation and non-negative constraints:

$$\sum_{p \in P(o,d,\tau)} r_{odp}^{\tau} = q_{od}^{\tau} \qquad r_{odp}^{\tau} \geq 0 \qquad \forall o, d, \tau \text{ and } p \in P(o, d, \tau)$$

TDUE Solution Algorithm (2/4)



TDUE Solution Algorithm (3/4)

- Solving the Restricted NMP (defined by the current subset of paths)
  - The route swapping based descent direction method
    - Given a feasible solution $r^l \in \Omega$ in an inner loop iteration l, obtain r^{l+1} as:

$$r_{odp}^{\tau,l+1} = \max\{0,\ r_{odp}^{\tau,l} \times [1-\rho^l \times \frac{c_{odp}^{\tau}(r^l) - \pi_{od}^{\tau}(r^l)}{c_{odp}^{\tau}(r^l)}]\} \quad \forall p \in \overline{P}(o,d,\tau) \setminus \overline{P}_{\pi}(o,d,\tau)$$

      $\overline{P}_{\pi}(o,d,\tau)$ is the referenced shortest path
    - Vehicles on the non-shortest paths are moved to the shortest path and the volume moved out from a non-shortest path is proportional to the relative difference in path cost between non-shortest paths and the shortest path.
    - A scaling factor helps to remedy the potential drawbacks of using absolute path-time differences in the path assignment updating. (Inver Hessian matrix in Newton-type methods).

TDUE Solution Algorithm (4/4)

- Vehicle-based Implementation Technique for Large Networks
  - The proposed DUE algorithm is path flow-based
    - Storage of path set and path assignment increases from iteration to iteration
    - Computer memory requirements grow dramatically when network is large
  - Particle-based traffic simulator
    - Individual vehicles are tracked and moved along their journeys.
    - Vehicle paths implicitly reflect the path set and path assignments results.
    - Large-scale network: number of OD pair > number of vehicles
  - Vehicle-based implementation
    - In iteration l, $(c(r)-\pi(r))/c(r)$ of vehicles are moved to the shortest path.
    - The vehicle path set as a proxy for the exact path set and assignment results

Numerical Experiments (1/7)

- DYNASMART – A Mesoscopic Traffic Simulator
  - $c(r)$: experienced path times or costs.
  - $Gap(r)$: vehicle experienced cost (VEC) or aggregated link cost (ALC) gap.
- Compare the two algorithms
  - CGDDM: Column Generation-based with the Descent Direction Method
  - CGMSA: Column Generation-based with the Method of Successive Averages (MSA: $r^{l+1} = r^l + 1/l \times (y^l - r^l) = (1-1/l) \times r^l + 1/l \times y^l$)
- Measure of effectiveness (MOE)
  - $Gap(r)$
  - $AGap(r)$

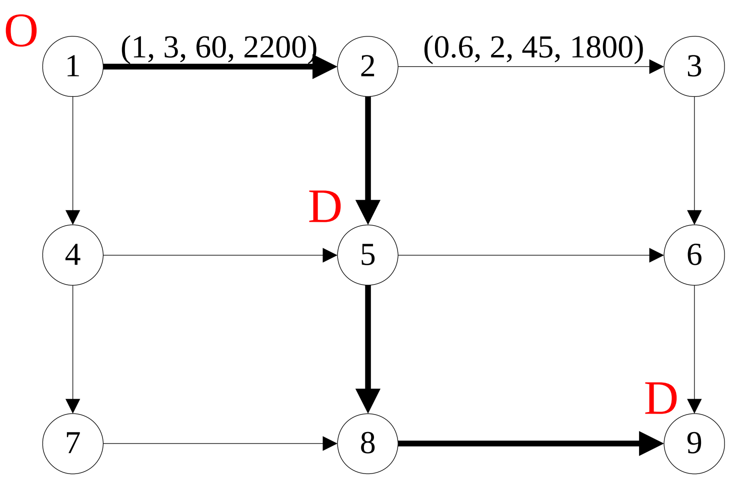
$$AGap(r) = \frac{\sum_{o\in O}\sum_{d\in D}\sum_{\tau\in T}\sum_{p\in P(o,d,\tau)} r_{odp}^{\tau}[c_{odp}^{\tau}(r) - \pi_{od}^{\tau}(r)]}{\sum_{o\in O}\sum_{d\in D}\sum_{\tau\in T}\sum_{p\in P(o,d,\tau)} r_{odp}^{\tau}}$$

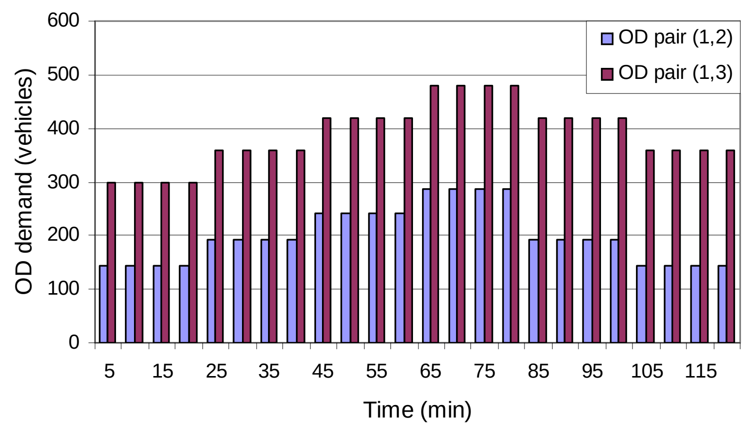
Numerical Experiments (2/7)

- A grid network with 2 OD pairs
  - CGMSA
  - CGDDM + 3 different step sizes

$$r_{odp}^{\tau,l+1} = \max\{0,\ r_{odp}^{\tau,l} \times [1 - \rho^{l} \times \frac{c_{odp}^{\tau}(r^{l}) - \pi_{od}^{\tau}(r^{l})}{c_{odp}^{\tau}(r^{l})}]\}$$

    - MSA step: $\rho^{l} = 1/(l+1), \forall\ l$.
    - Unit step: $\rho^{l} = 1, \forall\ l$.
    - Mixed step: $\rho^{l} = 1/k$, if $l = 0$; $\rho^{l} = 1$, otherwise.





Numerical Experiments (3/7)

- Convergence and Solution Quality

(K_{max} =20, L_{max} =10, assign. int. =1.0-min., initial gap =3649.4, and initial average gap =0.325)

Iteration counter k	vehicle experienced cost gap				average vehicle experienced cost gap			
	MSA step	CGDDM Unit step	CGDDM mix step	CGMSA	MSA step	CGDDM unit step	CGDDM mix step	CGMSA
1	1458.0	307.9	328.4	1046.9	0.130	0.027	0.029	0.093
2	1039.6	262.0	250.3	865.5	0.093	0.023	0.022	0.077
3	756.4	204.6	192.8	898.4	0.067	0.018	0.017	0.080
4	595.8	145.6	135.0	720.4	0.053	0.013	0.012	0.064
5	448.9	168.0	148.3	795.1	0.040	0.015	0.013	0.071
6	380.9	143.2	136.0	679.5	0.034	0.013	0.012	0.060
7	314.2	134.6	126.3	653.5	0.028	0.012	0.011	0.058
8			112.1	658.3			0.010	0.059
9			105.3	568.1			0.009	0.051
10				857.5				0.076
11				579.8				0.052
12				669.6				0.060
13				597.4				0.053
14				554.0				0.049

Numerical Experiments (4/7)

- CGDDM with mixed step size

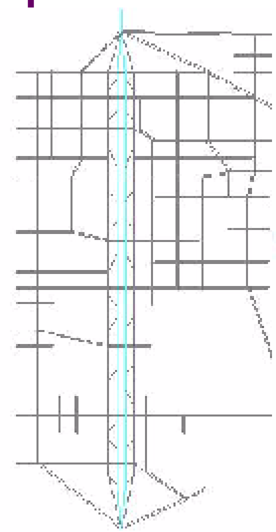
  (Mix step size, *Kmax* = 20, assign. int. = 1.0-min., experienced cost gap, and ini. gap = 3649.4)

L_{max}	1	2	3	5	10	15
Objective value (gap)	1072.9	647.3	273.6	232.6	105.3	106.4
Average Gap (min)	0.095	0.058	0.024	0.021	0.009	0.009
Reduction of initial gap (%)	70.6%	82.3%	92.5%	93.6%	97.1%	97.1%
Computation time (sec)	45 (13*)	62 (11)	80 (10)	124 (10)	213 (9)	302 (9)

- Summary of results on the grid test network
  - Able to find close-to-TDUE solutions on network with multiple OD pairs.
  - Outperform the commonly used heuristic (MSA).
  - No need to optimally solve the RNMP in the inner loop of the CGDDM.
  - The length of departure time interval does not affect the results.

Numerical Experiments (5/7)

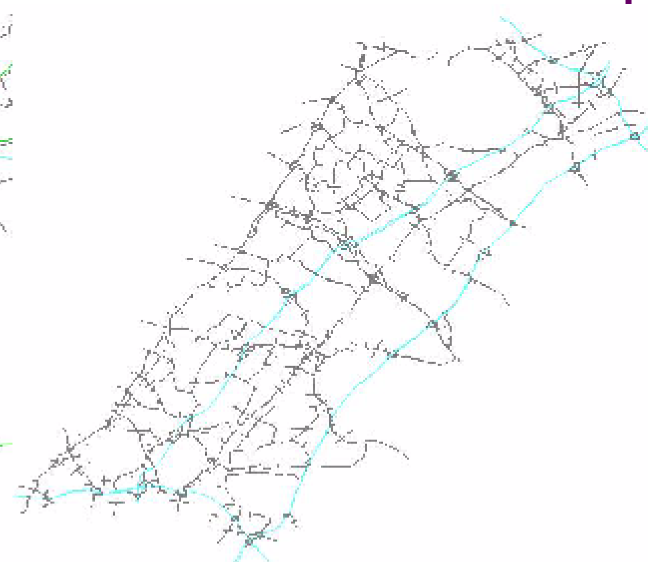
- The four large networks



(a) Fort-Worth network

(b) Irvine network

(c) Knoxville network



(d) Baltimore-Washington corridor network

Networks	# of zones	# of nodes	# of links	# of signals	# of vehicles in the observation period
Fort Worth, TX	13	180	445	62	27,447
Irvine, CA	61	326	626	70	35,304
Knoxville, TN	106	1347	3004	110	86,483
B-W, MD	111	2182	3387	231	91,389

Numerical Experiments (6/7)

- Solution quality on large networks - VEC Gap

	VEC Gap	Fort-Worth	Irvine	Knoxville	B-W corridor
Algorithms	Initial gap values	27463.3	1858.3	18444.9	38629.8
CGDDM with Mixed step	Gap(r*)	524.4	88.8	27.3	864.0
	AGap(r*)	0.019	0.003	0.000	0.009
	Gap Reduction (%)	98.1%	95.2%	99.8%	97.8%
CGDDM with Unit step	Gap(r*)	1833.8	148.7	128.4	2159.2
	AGap(r*)	0.067	0.004	0.001	0.024
	Gap Reduction (%)	93.3%	92.0%	99.3%	94.4%
CGDDM with MSA step	Gap(r*)	3075.8	327.6	385.9	4663.3
	AGap(r*)	0.112	0.009	0.004	0.051
	Gap Reduction (%)	88.8%	82.4%	97.9%	87.9%
CGMSA	Gap(r*)	4030.0	543.4	1186.2	11782.1
	AGap(r*)	0.147	0.015	0.014	0.129
	Gap Reduction (%)	85.3%	70.8%	93.6%	69.5%

Numerical Experiments (7/7)

- Solution quality on large networks - ALC Gap

	ALC Gap	Fort-Worth	Irvine	Knoxville	B-W corridor
Algorithms	Initial gap values	36110.0	3240.5	30384.3	41273.5
CGDDM	Gap(r*)	643.2	243.9	18.9	661.7
with	AGap(r*)	0.023	0.007	0.000	0.007
Mixed step	Gap Reduction (%)	98.2%	92.5%	99.9%	98.4%
CGDDM	Gap(r*)	2135.4	526.4	78.0	1534.4
with Unit	AGap(r*)	0.078	0.015	0.001	0.017
step	Gap Reduction (%)	94.1%	83.8%	99.7%	96.3%
CGDDM	Gap(r*)	4561.7	627.3	478.7	4728.4
with MSA	AGap(r*)	0.166	0.018	0.006	0.052
step	Gap Reduction (%)	87.4%	80.6%	98.4%	88.5%
CGMSA	Gap(r*)	7602.5	797.2	1876.4	10689.8
	AGap(r*)	0.277	0.023	0.022	0.117
	Gap Reduction (%)	78.9%	75.4%	93.8%	74.1%


THE
END


OR GOES TO
WASHINGTON