Transit Network Design under Stochastic Demand

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Background

- **Public transport**
  - **Fixed route transit (FRT):** fixed route and fixed schedule, large capacity, exclusive right of way, such as metro, bus, or regular ferry
  - **Demand responsive transit (DRT):** flexible route, small or medium vehicle, complementary to FRT, such as ridesharing services, taxi, etc.

- **Transit Network Design Problem (TNDP)**
  - Find a most cost efficient operating plan for FRT while satisfying passenger demand

- **Challenges in TNDP**
  - Jointly optimize the FRT and DRT as an integrated system under stochastic demand, hard capacity constraints, and user equilibrium flows
Background

Schedule-based TNDP
- Time-space network
- Decision variables: route, schedule, fleet size
- Integer (a lot) and real variables

Frequency-based TNDP
- Objective: minimize construction and passenger cost
- Decision variables: route alignment, service frequency
- Integer and real variables
Background

- **Deterministic TNDP**
  - Given OD matrix
  - First formulated by Magnanti (1984) as a mixed integer linear program (MILP)

- **Stochastic TNDP**
  - Stochastic programming approach (Ruszczynski, 2008)
    - Stochastic demand follows a known probability distribution
    - Monte-Carlo simulation to approximate the cost expectation by sample average, two-stage stochastic problem

- **Robust optimization approach**
  - Stochastic demand captured by an uncertainty set
  - Min-max problem, worst case scenario (Ben-tal et al., 2004)
Background

- Solution algorithm for stochastic approach
  - Exact method: multi-dimensional integral evaluation, formidable task
  - Heuristic approach: search the neighborhood of the initial solution, efficient but solution quality not guaranteed (Hoff et al. 2011)
  - Approximation method: L-shaped/ Multi-cut method, long computation time, global optimal solution is not guaranteed
- Solution algorithm for robust optimization approach
  - Polyhedral uncertainty set, linearization
  - Same dimension as its deterministic counterpart
  - Conservative, dependent on the size of the uncertainty set
Objectives

- To develop a modeling framework for combining FRT and DRT network design under stochastic demand
  - Investigate the benefits of the integrated services under stochastic demand
  - Develop a service reliability (SR) based formulation and solution algorithm to address demand uncertainty
- To assess the performance of SR-based formulation
  - Two application contexts: ferry network and rapid transit network
  - Two passenger flow distribution pattern: system optimal (SO) and user equilibrium (UE)
Transit Network Design with Stochastic Demand under System Optimal Flows

Time-space network description

➢ **Ferry** time-space network

![Diagram of Ferry time-space network]

➢ **Passenger** time-space network

![Diagram of Passenger time-space network]

➢ **Notations**

\[
Y = \{Y_{ij}, ij \in \left[ S^f \cup W^f \right] \} \quad \text{Ferry service arc}
\]

\[
X = \{X_{ij}^d, ij \in \left[ S^d \cup W^d \right] \} \quad \text{Passenger service arc}
\]
Research objective

Objectives

- Capture demand uncertainty on ferry service deployment
- Develop a modeling framework for combining FRT and DRT

Regular ferry services (FRT)
- Fixed schedule
- Large capacity
- Low unit cost

Ad-hoc ferry services (DRT)
- Flexible schedule
- Small capacity
- High unit cost
Objective:

- Regular Services cost
- Ad-hoc services cost
- Passenger cost

Decisions:

- The deployment of these two services are related
- Two-stage stochastic program
- Represent stochastic demand by a large number of discrete scenarios
- A large size MILP

The original problem
Service reliability $\rho$

- The probability of passengers carried by regular ferry services
- A vector, one for an OD pair

<table>
<thead>
<tr>
<th>Service reliability</th>
<th>Regular service</th>
<th>Ad-hoc service</th>
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<tr>
<td>low</td>
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</table>

Tradeoff

$\rho_1 = 0.3$

$\rho_2 = 0.7$
SR-based stochastic formulation (Phase-1)

**Phase-1**

regular service deployment

\[
\min_Y \sum_{i \in N_b^J} \sum_{j \in N^J \setminus N_b^J} Y_{ij} c_{ij} + \sum_{ij \in S^J} Y_{ij} c_{ij}^F
\]

- **Fixed cost**
- **Operating cost**

(1) Passenger flow conservation

\[
\sum_{j \in N^d} X_{ij}^d - \sum_{k \in N^d} X_{ki}^d = \begin{cases}
\bar{B}^d & \text{if } i \text{ is the O of } OD \ d \\
-\bar{B}^d & \text{if } i \text{ is the D of } OD \ d \\
0 & \text{otherwise}
\end{cases} \quad \forall i \in N^d
\]

(2) Service reliability constraint

\[
\bar{B}^d = \Psi^{-1}_d (\rho^d)
\]

\[
0 \leq \rho^d \leq 1 \quad \forall d
\]

Stochastic demand

Demand realized within the service reliability boundary can be carried by regular ferry services
SR-based stochastic formulation (Phase-1)

\[
\min_Y \sum_{i \in N^f_b} \sum_{j \in N^f \setminus N^f_b} Y_{ij} c^1_{ij} + \sum_{ij \in S^f} Y_{ij} c^1_{ij}
\]

\[
\sum_{j \in N} Y_{ij} - \sum_{k \in N} Y_{ki} = 0 \quad \forall i \in N
\]

(3)

\[
0 \leq Y_{ij} \leq U_{ij}
\]

\[
Y_{ij} \in \text{integer} \quad \forall ij \in A
\]

(4)

\[
\sum_{i \in N^f_b} \sum_{j \in N^f \setminus N^f_b} Y_{ij} \leq V
\]

(5)

\[
\sum_{d \in R} X^d_{ij} \leq Y_{ij} \zeta \quad \forall ij \in S^f
\]

Regular services connection constraints

Fleet size constraint

Capacity constraints
SR-based stochastic formulation (Phase-2)

Phase-2 Ad-hoc services deployment $Z_e$

\[ Z_e \min \quad \bar{Q}(\rho) = \sum_{e \in E} p_e \left( \sum_{d \in R} c_d^3 Z_d^e + \sum_{d \in R} \sum_{ij \in A} c_{ij}^2 X_{ij,e}^d \right) \]

Ad-hoc cost \quad Passenger cost

1. \begin{align*}
\sum_{j \in N} X_{kj,e}^d - \sum_{i \in N} X_{ik,e}^d &= B_e^d - Z_e^d \quad \text{if $k$ is the O of OD $d$} \\
\sum_{j \in N} X_{kj,e}^d - \sum_{i \in N} X_{ik,e}^d &= Z_e^d - B_e^d \quad \text{if $k$ is the D of OD $d$, } \forall k \in N, d \in R \\
\sum_{j \in N} X_{kj,e}^d &= 0 \quad \text{otherwise}
\end{align*}

Passenger flow conservation

Capacity constraints

2. \[ \sum_{d \in R} X_{ij,e}^d \leq Y_{ij} \quad \forall ij \in S^f \]

Stochastic demand

Demand beyond the service reliability is to be carried by ad-hoc services
SR-based gradient approach

Find the optimal service reliability $\rho$

Phase-1  Phase-2  Total cost

FRT  DRT

Find the gradient of total cost w.r.t $\rho$

$\begin{pmatrix}
\rho^1 \\
\rho^2 \\
\vdots \\
\rho^d
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\rho^1 + \Delta \rho^1 \\
\rho^2 \\
\vdots \\
\rho^d
\end{pmatrix}$

$\rho'$

increase $\rho$ by a small number each time until $Y$ changes

$\Delta g 
\rightarrow
\Delta Q 
\rightarrow
\Delta \phi(\rho) = \Delta g + \Delta Q$

$\partial \phi(\rho) \approx \frac{\Delta \phi(\rho)}{\Delta \rho^1}$

Disadvantage: computation time highly depends on the number of OD pairs $d$
Advantage of the SR-based model

- Take advantage of the special structure of the problem
- Separate the large size MILP into one smaller size MILP and one LP
- Can be extended to include the user equilibrium assignment principle
Transit Network Design with Stochastic Demand under User Equilibrium Flows

Scheme A (with passenger reservation)

- Allow advance reservation for passengers
  - Passenger demand for the next day is obtained one-day in advance
  - Provide sufficient amount of ad-hoc services considering user equilibrium

Demand realization $b_1=300$, $b_2=300$, regular service capacity= 250, 500
no need to provide ad-hoc services, reduce operating cost
User equilibrium with stochastic demand

- Not sensible to find the long-term UE as demand varies from day to day
- Find the short-term UE: for a demand realization, same minimum traveling cost for passengers on the same origin-destination (OD)
- Passenger options: take the congested direct service, wait for the next direct service or take a detour

Travelling cost = waiting + on vehicle + overflow delay
Stochastic Formulation under UE (Phase-1)

Phase-1

Regular ferry services routes and schedule $Y$

The same as SO

\[
\min_Y \sum_{i \in N^f_b} \sum_{j \in N^f \setminus N^f_b} Y_{ij} c_{ij}^F + \sum_{ij \in S^f} Y_{ij} c_{ij}
\]

Regular services cost

\[
\sum_{j \in N^d} X_{ij}^d - \sum_{k \in N^d} X_{ki}^d = \begin{cases} 
B^d_i & \text{if } i \text{ is the O of } OD \ d \\
-B^d_i & \text{if } i \text{ is the D of } OD \ d \\
0 & \text{otherwise}
\end{cases} \quad \forall i \in N^d
\]

1) Regular services connectivity constraints
2) Passenger flow conservation constraints
3) Capacity constraints
**Stochastic Formulation under UE (Phase-2)**

**Phase-2**

Dynamic ad-hoc services deployment $Z_e$

For a certain ad-hoc services cost $\theta_e, \min \leq \theta_e \leq \theta_e, \max$

$$
\min_{Z_e} \quad \bar{Q}(\rho) = \sum_{e \in E} p_e \left( \sum_d c_d^3 Z_e^d + \sum_{ij \in A} c_{ij}^2 \sum_d X_{ij,e}^d + \sum_{ij \in A} -\beta_{ij,e} \left( \sum_d X_{ij,e}^d \right) \right)
$$

- **Ad-hoc cost**
- **Travel time**
- **Overflow delay**
- **Passenger cost**

(P4.2) For a certain ad-hoc services cost $\theta_e, \min \leq \theta_e \leq \theta_e, \max$

$$
\min_{X,Z} \quad f = \sum_{ij \in A} c_{ij}^2 \sum_d X_{ij,e}^d
$$

Travel time cost

$$
\sum_{j \in N} X_{kj,e}^d - \sum_{i \in N} X_{ik,e}^d = \begin{cases} B_e^d - Z_e^d, & \text{if } k \text{ is the origin of OD } d \\ 0, & \text{otherwise} \end{cases}, \forall k \in N
$$

Overflow delay

$$
\sum_d X_{ij,e}^d \leq \zeta Y_{ij}, \forall ij \in A
$$

Overflow delay

$$
\sum_{d \in D} c_d^4 Z_e^d = \theta_e
$$

Ad-hoc cost

**Proposition 4.1:** P4.2 yields a UE flow pattern under capacity constraints, with the negative Lagrange multiplier associated with the link capacity constraint representing the corresponding passenger overflow delay.
SR-based gradient solution procedure

- **Phase-1**: Regular ferry services routes and schedule
- **Phase-2**: Dynamic ad-hoc deployment
- **Total cost**: Phase-1 + Phase-2 cost

Find the optimal service reliability

Find the optimal ad-hoc services provision level

Increase θ by κ each time and the one with the lowest cost is kept as the optimal solution
Scheme B (no advance reservation)

- Passenger demand is revealed only when they arrive at the piers
- Provide ad-hoc services whenever there is demand overflow

Demand realization \( b_1 = 300, b_2 = 300 \), provide ad-hoc services 50, spaces wasted for \( B_2 \)
Scheme B (no advance reservation)

- No waiting or detour, only direct service, best for passengers
- Most costly plan for the company

Travelling time = on vehicle travel time
SR-based stochastic formulation
(single phase)

- **Phase-1**: Regular ferry services routes and schedule \( \mathbf{Y} \)
  - Up to demand \( B_d = \Psi_d^{-1}(\rho^d) \)

- **Phase-2**: Dynamic ad-hoc deployment \( Z_{d,e} = \max\{0, B_{d,e} - B_d\} \)
  - Expected ad-hoc services cost can be calculated by \( \bar{\theta}(\rho) = \sum_d \int_{B_{d,e} \geq B_d} c^3 Z_{d,e} \Psi_d \, de \)

- **Single phase problem**

\[
\min_{h, \mathbf{Y}, \rho} \phi(\rho) = (c^1)^T \mathbf{Y} + \bar{\theta} + (c^4)^T \mathbf{B}
\]

<table>
<thead>
<tr>
<th>Regular service cost</th>
<th>Ad-hoc service cost</th>
<th>Passenger cost</th>
</tr>
</thead>
</table>

s.t. Regular services connection constraints
Fleet size constraint
Capacity constraints
Passenger flow conservation

- **Value of reservation**: cost difference between scheme A and B
Numerical study

Ferry service network in Hong Kong

- Studying horizon: 7am-9am
- Time interval: 15 min

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<th>MW-CBD</th>
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</table>
Solution procedure illustration

- Different starting points comparison
- Computation time: SR-based method: 217 seconds versus L-shaped method: 6 hours

![Graph showing cost and starting points comparison](image-url)
Result analysis

- Group: different unit ad-hoc cost and COV (coefficient of variation)
- Total company cost under SO is lower than that under UE
- The value of reservation between scheme A and B increase with COV

Service deployment comparison between UE and SO solutions
Robust Rapid Transit Network Design

Research objectives

- Rapid TNDP: location of stations, route alignment and frequency

- Objectives
  - Investigate the TNDP by robust optimization
  - The optimal level of robustness to minimize the system cost

- Methodology
  - Apply the SR-based two-phase formulation
  - Improve the efficiency of the gradient solution procedure
Robust optimization to address demand uncertainty

- Robust optimization
  - Day to day variation in demand, min-max problem, worst case scenario (Ben-tal et al., 2004)
  - Conservative solutions
- Evaluate the outcome of uncertainty set
  - Search for the optimal robustness level to hedge against uncertainty
  - Demand fluctuation, uneconomical to rely on transit lines alone

Stochastic demand

Uncertainty set

system cost vs. robustness level
Service reliability to address demand uncertainty

- Service reliability $\rho$: the probability that passengers can be carried by rapid transit services
- Conveys the level of robustness, size of the uncertainty set
- Dial-a-ride services cost evaluate the outcomes beyond the uncertainty set

\[ \rho_1 = 0.3 \]
Network representation

- Multi-line design: dummy origin and destination
- Passenger transfer and waiting: line sub node and station sub node

- Dummy O and D: dark gray node
- Dummy arcs: black dashed line
- Transfer time: blue line
- Get on/off time: green line
- On vehicle travel time: orange line
- Demand: imposed on the station sub node
Two-phase Robust Formulation under System Optimal Flows
Two phase robust formulation

**Phase-1**
Rapid Transit line (RTL) alignment and frequencies $W, Y, f$

$$
\min_{W,Y,f} \sum_{r \in R} \sum_{ij \in A} c_{ij}^1 f_r Y_{ij}^r + \sum_{r \in R} \sum_{ij \in A} c_{ij}^2 Y_{ij}^r + \sum_{i \in N} c_i^3 W_i
$$

Transit operating  | Line construction  | Station construction

**Phase-2**
Dynamic dial-a-ride services deployment $Z_e$

$$
\min_{Z_e} \overline{Q}(\rho) = \sum_{h \in H} p_e \left( \sum_{ij \in A} c_{ij}^d Z_{ij}^d \right) + \sum_{ij \in A} \sum_{d \in D} X_{ij,c}^d
$$

Dial-a-ride cost  | Passenger cost

- **Total cost** = Phase-1 + Phase-2 cost

**Decision variables:**
- $W$: Station location
- $Y$: Route alignment
- $f$: Frequency
- $Z$: Dial-a-ride services
Solution algorithm: find the decent direction (revised)

\[ \min_{f,Y} g = \sum_{r \in R} \sum_{i,j \in A} \left( c_{ij} f_r Y_{ij} + c_{ij}^2 Y_{ij}^r \right) + \sum_{i \in N} c_{ij} W_i \]

\[ \varphi(\rho) = g + \bar{Q}(\rho) \]

\[ \begin{align*}
\rho' & = (\rho^1 + \Delta \rho^1, \rho^2, \ldots, \rho^d) \\
\Delta g & = \frac{1}{h} \sum_{h} \sum_{i,j} \left( \beta_{ij,h} \right) \sum_{r} C \left( Y_{ij}^r + Y_{ji}^r \right) \Delta f_r \rightarrow \Delta \bar{Q}(\rho)
\end{align*} \]

Advantages: computation time does not depend on the number of OD pairs

Assumption: A small perturbation in \( \rho \) changes frequency \( f \) only while maintaining the line alignment \( Y \)
Two Phase Robust Formulation under User Equilibrium Flows
Two-phase robust formulation

- Short term UE with stochastic demand
  - Passenger options: different routes choices on transit line
  - Over flow delay: transit services operated at capacity
  - Passenger travelling cost = on-vehicle travel time + overflow delay

Phase-1: Feasible region for $\rho \in [0,1]$

Phase-2: Dial-a-ride provision $\theta_e \in [\theta_{e,\min}, \theta_{e,\max}]$, $e$ for one demand realization

(P6.4): Total required dial-a-ride services $\theta_{e,\min}$ just enough to carry demand overflow
Lower bound $\min_Z \theta_{e,\min} = (c^3)^T Z_e$

(P6.2): Maximum dial-a-ride services cost $\theta_{e,\max}$
Upper bound When the overflow delay is zero, every passenger can take the shortest path
Solution procedure

(P6.3): Find the optimal dial-a-ride services provision level \( \theta_{e,\text{min}} \leq \theta_e \leq \theta_{e,\text{max}} \)

Detect the feasibility range: \( [\theta_e, \bar{\theta}_e] \)

Advantage: (1) be able to find the exact optimal solution
(2) reduce the computation time to 1/10
Case study

- Three lines were generated
- Frequency SO: \( f_1 = 16, f_2 = 13, f_3 = 7 \) vehicle/h

Route 1: 9 — 447 — 8 — 802 — 2 — 840 — 1 — 393 — 6 — 393 — 5

Route 2: 4 — 705 — 1 — 955 — 7

Route 3: 3 — 505 — 2 — 200 — 10

- Frequency UE: \( f_1 = 19, f_2 = 16, f_3 = 7 \) vehicle/h

Route 1: 9 — 882 — 2 — 859 — 1 — 659 — 6 — 409 — 5

Route 2: 4 — 772 — 1 — 772 — 7

Route 3: 8 — 368 — 9 — 418 — 10 — 368 — 3
Result analysis

- RTL carries most of the passengers while dial-a-ride serves as a supplementary role
- UE requires more dial-a-ride services to bring down the overflow delay cost

<table>
<thead>
<tr>
<th>OD pair</th>
<th>OD demand</th>
<th>Path by RTL</th>
<th>Expected RTL Patronage</th>
<th>Transfer</th>
<th>Distance by RTL</th>
<th>Path by RTL</th>
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91 % 87 %
Result analysis

- The SR-based approach obtains the optimal level of robustness with lowest total cost.
- Higher level of robustness indicates higher transit cost and lower Dial-a-ride cost.

<table>
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<th>Level of robustness</th>
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<th>Passenger cost</th>
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<table>
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<th>Level of robustness</th>
<th>Company cost</th>
<th>Passenger cost</th>
<th>Total cost</th>
</tr>
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<tr>
<td></td>
<td></td>
<td>RTL</td>
<td>Dial-a-ride</td>
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<td>SR-based method</td>
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<td>697</td>
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<tr>
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<td>2 * σ</td>
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<td>1346</td>
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<td>3 * σ</td>
<td>0.998</td>
<td>1455</td>
<td>11</td>
<td>736</td>
</tr>
</tbody>
</table>
(a) Solutions under SO

(b) Solutions under UE
Conclusions

- **Schedule based ferry service network design problem under demand uncertainty**
  - Combination of regular ferry services and ad-hoc services
  - Service reliability to separate the two service types deployment into two phases
  - Two passenger flow patterns SO and UE are investigated
  - Value of reservation to the company under stochastic demand

- **Frequency based rapid transit network design problem**
  - Multi-line design without predetermined origin and destination
  - Transfer costs are accounted for
  - Consider the problem from the perspective of robust optimization
  - Service reliability conveys the level of robustness
  - Improve the current SR based gradient solution algorithm
Key Takeways

• Jointly optimize FRT and DRT as an integrated system can achieve substantial cost efficiency in addressing stochastic demand.

• The notion of Service Reliability (SR) offers an efficient way to reformulate and solve stochastic programs, which also allows the incorporation of three difficult extensions in TND: addressing stochastic demand, hard capacity constraints, and user equilibrium conditions.