Modeling Driver Behavior as a Hazard-Based Risk-Taking Process

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Recent Developments in Microscopic Traffic Simulation
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Introduction: Motivation/Framework/Objective/Approach

Background: Assessment of Major Lane-Changing/Acceleration Models

The Model:
- Tactical Stage:
  - Duration (Hazard-Based) Models Theoretical Framework
  - Data Description and Extraction
  - Estimation Statistics
- Operational Stage
  - Formulation Logic
  - Model Implementation: Asymptotic Expansion
  - Sensitivity Analysis: Initial Plots

Numerical Results: Fundamental Diagram

Conclusion
Motivation (Micro-Crash Level):

In 2006, the US. Department of Transportation reported 42,624 traffic crash fatalities with 2,575,000 injuries and 3,014,116,000,000 vehicle-miles traveled.
Motivation (Micro-Crash Level):

Plotting the crash distribution by time of day over the last 8 years:

(U.S. Department of Transportation, National Highway Traffic Safety Administration, Nov. 2006)
### Motivation (Macro – Disaster Level):

<table>
<thead>
<tr>
<th>Some Recent Extreme Condition</th>
<th>Date</th>
<th>Countries Affected</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York September Attack</td>
<td>September the 11th, 2001</td>
<td>United States</td>
</tr>
<tr>
<td>Indian Ocean Earthquake/Tsunami</td>
<td>December the 26th, 2004</td>
<td>Indonesia,  Sri Lanka, India</td>
</tr>
<tr>
<td>London Bombing</td>
<td>July the 7th, 2005</td>
<td>England</td>
</tr>
<tr>
<td>Hurricane Katrina</td>
<td>August the 29th, 2005</td>
<td>United States</td>
</tr>
<tr>
<td>2008 New York Explosion</td>
<td>March the 6th, 2008</td>
<td>United States</td>
</tr>
<tr>
<td>Iowa/Wisconsin Flooding</td>
<td>June, 2008</td>
<td>United States</td>
</tr>
</tbody>
</table>

**Outline**
- Introduction
- Background
- The Model: Tactical Stage
- Operational Stage
- Numerical Results
- Conclusion

**Diagram:**
- Timeline from 2001 to 2008
  - Key events and their respective years
  - Visual representations of some disasters (e.g., New York September Attack, Indian Ocean Earthquake/Tsunami, London Bombing, Hurricane Katrina, 2008 New York Explosion, Iowa/Wisconsin Flooding)
Framework: Drivers Decision Models

- Pre-Trip
- Strategic En-Route
- Tactical Route Execution
- Operational Driving
- Vehicle Control

Focus

Time to Make and Execute Decision:
- ~ 1 hour
- 30 seconds
- 5 seconds
- Instantaneous

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FHWA, 2004
Motivation:

1- Existing car-following and lane-changing models are accident-free where safety constraints are forced.

2- There is a need for a richer complete representation of the cognitive processes underlying driver behavior in different driving conditions (free-flow, congested and extreme conditions).

Objectives:

1- incorporate cognitive dimensions of the driver task in microscopic traffic simulation

2- include risk-perception and risk-taking behaviors under uncertainty (stochastically)

3- capture driver behavior in complex environments such as those associated with congested conditions, accident-prone situations and extreme regimes.
Background: Car-Following Models/Lane Changing Models
Major car-following models:

1. GHR (GM) (Gazis, Herman and Rothery, 1959)
2. Gipps (Gipps, 1981)
3. CA (Nagel and Shreckenberg, 1992) / (Krauss et al., 1996)
4. SK (Krauss and Wagner, 1997)
5. IDM (Treiber et al., 2000)
6. IDMM (Treiber and Helbing, 2003)
7. Wiedemann (Wiedemann, 1974)

Major lane-changing models

1. Gipps Model (Gipps, 1986)
2. Wiedemann Model (Wiedemann, 1991)
4. Hidas Model (Hidas, 2002)
Model Formulation
Modeling Framework

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Vehicle $n$ of Interest Free-Flow Episode
Stopped Vehicle Car-Following Episode
Slow Vehicle Acceleration Update Function for Free-Flow/Car-Following Model
Lane Changing Model: Theoretical Framework

(Hamdar and Mahmassani, 2008)
Baseline hazard (1)

- Let $T_i$ be a non-negative random variable representing the duration of car-following/free-flow episode (time) of a driver $i$.

- The hazard at time $u$, $\lambda_i(u)$, is the instantaneous probability that the car-following/free-flow duration $T_i$ for a driver $i$ will end in an infinitesimally small time period $\delta$ after time $u$, given that the duration has elapsed until time $u$:

$$
\lambda_i(u) = \lim_{\delta \to 0^+} \frac{\Pr(u \leq T_i < u + \delta \mid T_i \geq u)}{\delta}
$$

- We can relate the hazard to the density function $f_i(.)$ and the cumulative distribution function $F_i(.)$ of $T_i$ by:

$$
\lambda_i(u) = \frac{f_i(u)}{1 - F_i(u)} = \frac{f_i(u)}{S_i(u)}
$$

Endurance or Survival Probability
Baseline hazard (2):
Focus on Three Parametric Functions

1. Simplest assumption: constant hazard rate \( \lambda_i(u) = \sigma \) implying that there is no duration dependence
   duration time \( T_i \) exponentially distributed

2. Monotonic duration dependence-- two-parameter hazard function:
   \[ \lambda_i(u) = \sigma \alpha (\sigma u)^{-\alpha - 1} \]
   where \( \sigma > 0 \) and \( \alpha > 0 \)
   - \( \alpha > 1 \): monotonically increasing dependence (snowballing effect)
   - \( \alpha < 1 \): monotonically decreasing (inertia effect) dependence
   - \( \alpha = 0 \): no duration dependence.
   distribution of \( T_i \) is weibull
Baseline hazard (3): Focus on Three Parametric Functions

3. **Non-monotonic duration dependence**—the hazard function has the following form:

\[ \lambda_i(u) = \frac{\sigma\alpha(\sigma u)^{\alpha-1}}{1 + ((\sigma u)^\alpha)} \]

- hazard function is monotonic decreasing from infinity when $\alpha < 1$
- monotonic decreasing from $\sigma$ when $\alpha = 1$
- increasing from zero to a maximum of
  \[ u = \left(\frac{1}{\alpha-1}\right)^{1/\alpha} / \sigma \]
  and decreasing thereafter when $\alpha > 1$

Distribution of $T_i$ is Log-Logistic
Effect of External Covariates: Proportional Hazard Form

- The advantage of this method is that it can take into account the inter-drivers’ heterogeneity (assumed a gamma heterogeneity).

- The external covariates are multiplicative on an underlying baseline hazard function:

\[ \lambda_i(u, x_i, \beta, \lambda_0) = \lambda_0(u) \phi(x_i, \beta) \]

Where
- \( \lambda_0(u) \) = baseline hazard at time \( u \)
- \( x_i \) = vector of explanatory variables corresponding to driver \( i \)
- \( \beta \) = vector of parameters corresponding to \( x_i \) to be estimated

- For ease of estimation: \( \phi(x_i, \beta) = \exp(-\beta' x_i + w_i) \) ; The term \( w_i \) represents the unobserved heterogeneity
As defined by the authors, driving episodes are of two types: free-flow episode versus car-following episodes.

A vehicle exits a “free-flow” episode when:
- it changes lanes (Episode Type 3, \( q = 4 \)). This is rarely expected – unless it is a mandatory lane change towards desired exit.
- it becomes close enough to the leader entering another “car-following” episode (Episode Type 4, \( q = 3 \)).

As for the “car-following episodes”, they end when:
- the corresponding vehicle changes lanes (Episode Type 1, \( q = 2 \)).
- When the leader of the vehicle of interest is far-enough so it enters a free-flow episode (Episode Type 2, \( q = 1 \)).
Driving: a Multiple Duration Process (2)

- Two key questions in the lane-changing model
  - Is it desirable to change lanes?
  - Is it possible to change lanes?

Captured by the Hazard-Function Above (q = 1; q = 3)

Boundary Conditions/MOBIL Model (Treiber et al., 2007)
Lane-Changing Model: Data Description and Extraction
The data set is trajectory data for 5678 vehicles collected as a part of the Federal Highway Administration’s (FHWA) Next Generation Simulation Simulation (NGSIM) project.

Data were collected on the 13th of April, 2005, between 4 PM and 5:30 PM, on a segment of the Interstate I-80 in Emeryville, San Francisco, USA.

The semi-poisson headway distribution model (Wasielewski, 1974) is used to determine the threshold $T_{\text{critical}}$ below which a car-following behavior is assumed and above which a free-flow behavior is assumed.
### Exogenous Covariates Included in the Study

<table>
<thead>
<tr>
<th>Exogenous Covariate</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$ (LCL)</td>
<td>Number of Leaders Changing Lanes During Episode</td>
</tr>
<tr>
<td>$X_2$ (V)</td>
<td>driver’s speed</td>
</tr>
<tr>
<td>$X_3$ (DXL1)</td>
<td>headway between driver $i$ and leader $i-1$ (front-to-front bumper).</td>
</tr>
<tr>
<td>$X_4$ (DVL1)</td>
<td>relative speed between driver $i$ and leader $i-1$.</td>
</tr>
<tr>
<td>$X_5$ (DXF1)</td>
<td>distance headway between driver $i$ and follower $i+1$ (front-to-front bumper).</td>
</tr>
<tr>
<td>$X_6$ (DVF1)</td>
<td>relative speed between driver $i$ and follower $i+1$.</td>
</tr>
<tr>
<td>$X_7$ (DXL2)</td>
<td>distance headway between driver $i$ and driver $i-2$ (front-to-front bumper).</td>
</tr>
<tr>
<td>$X_8$ (DVL2)</td>
<td>relative speed between driver $i$ and driver $i-2$.</td>
</tr>
<tr>
<td>$X_9$ (DXL1R)</td>
<td>distance headway between driver $i$ and the corresponding leader on the right lane.</td>
</tr>
<tr>
<td>$X_{10}$ (DVL1R)</td>
<td>relative speed between driver $i$ and the corresponding leader on the right lane.</td>
</tr>
<tr>
<td>$X_{11}$ (DXF1R)</td>
<td>distance headway between driver $i$ and the corresponding follower on the right lane.</td>
</tr>
<tr>
<td>$X_{12}$ (DVF1R)</td>
<td>relative speed between driver $i$ and the corresponding follower on the right lane.</td>
</tr>
<tr>
<td>$X_{13}$ (DXL1L)</td>
<td>distance headway between driver $i$ and the corresponding leader on the left lane.</td>
</tr>
<tr>
<td>$X_{14}$ (DVL1L)</td>
<td>relative speed between driver $i$ and the corresponding leader on the left lane.</td>
</tr>
<tr>
<td>$X_{15}$ (DXF1L)</td>
<td>distance headway between driver $i$ and the corresponding follower on the left lane.</td>
</tr>
<tr>
<td>$X_{16}$ (DVF1L)</td>
<td>relative speed between driver $i$ and the corresponding follower on the left lane.</td>
</tr>
<tr>
<td>$X_{17}$ (K)</td>
<td>Driver’s average surrounding density. It is defined as the total density (over the number of lanes).</td>
</tr>
<tr>
<td>$X_{18}$ (KR)</td>
<td>Driver’s average surrounding density in adjacent lane 1 (to the right) if available.</td>
</tr>
<tr>
<td>$X_{19}$ (KL)</td>
<td>Driver’s average surrounding density in adjacent lane 2 (to the left) if available.</td>
</tr>
</tbody>
</table>
Lane-Changing Model: Testing Results
Minor Multiple-Duration Effect:

- Not surprisingly, using the calculated $T_{critical}$ thresholds, over 90% of the headways observed are in the car-following mode.

- This is expected since the data were collected at peak hour PM on a major free-way section: drivers are leaving the business districts of San-Francisco, San Jose (through the Bay Bridge) as well as Oakland for the residential areas in the Northern Part of the Bay Area.

Accordingly, all episodes are car-following episodes ending either by changing lanes or by leaving the study area.

- The multiple duration effect is minor; in this study, the duration model is a descriptive lane-changing model.
General Statistics:

- 6450 Episodes:
  - 4465 are left censored and thus cannot be supported by LIMDEP Software in the estimation process.
  - 1250 right censored episodes
  - 735 non-censored ones.

- For the data description, the non-censored episodes (735) were used:

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Duration Model (1): Weibull

<p>| Parameters of underlying density at data means: |</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sigma</td>
<td>0.1858</td>
<td>0.00088</td>
<td>0.0169 to 0.0203</td>
</tr>
<tr>
<td>Alpha</td>
<td>2.09356</td>
<td>0.12276</td>
<td>1.8529 to 2.3342</td>
</tr>
<tr>
<td>Median</td>
<td>48.22723</td>
<td>2.29176</td>
<td>43.7354 to 52.7191</td>
</tr>
</tbody>
</table>

| Percentiles of survival distribution: |
| Survival | 0.25 | 0.58 | 0.75 | 0.95 |
| Time      | 71.90 | 48.23 | 30.49 | 13.09 |

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Std.Err.</th>
<th>t-ratio</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE</td>
<td>4.227</td>
<td>35.393</td>
<td>2.89E-15</td>
</tr>
<tr>
<td>LCL</td>
<td>0.15396</td>
<td>7.93427</td>
<td>2.89E-15</td>
</tr>
<tr>
<td>V</td>
<td>-0.06056</td>
<td>-17.0549</td>
<td>2.89E-15</td>
</tr>
<tr>
<td>DXL1</td>
<td>-0.00149</td>
<td>-0.74179</td>
<td>0.458214</td>
</tr>
<tr>
<td>DVL1</td>
<td>0.010849</td>
<td>1.23447</td>
<td>0.217029</td>
</tr>
<tr>
<td>DXF1</td>
<td>0.001557</td>
<td>2.5156</td>
<td>0.011883</td>
</tr>
<tr>
<td>DVF1</td>
<td>0.010988</td>
<td>1.62369</td>
<td>0.104442</td>
</tr>
<tr>
<td>DXL2</td>
<td>0.008191</td>
<td>5.56678</td>
<td>2.59E-08</td>
</tr>
<tr>
<td>DVL2</td>
<td>0.012889</td>
<td>1.86684</td>
<td>0.061924</td>
</tr>
</tbody>
</table>

>1 (2.09): snow-balling effect. This is a sign of an increase of impatience level while driving in congested conditions.

The statistically significant variables are LCL, V, DXL2, DVL2 and DXF1. The variables related to followers (anisotropy of traffic) and second leaders (anticipation) have a significant influence on the probability of ending an episode and thus, changing lanes.
Interestingly enough, the remarks regarding the external covariates in the Weibull Model remains the same in the Log-Logistic Model. The only difference is adding the density as a significant external covariate.
Duration Model (3): Log-logistic

- $\alpha$ is greater than 1 (2.5)

- The hazard function increases with the duration until the duration is equal to $u = \left(\frac{\alpha-1}{\alpha}\right)^{1/\alpha}/\sigma$. The probability of ending an episode (changing lanes) starts decreasing afterward. This may reflect the driver being used to travel on a given lane after a specific amount of time.

- Behavior is comparable to that captured by Weibull Model, up to the $u = \left(\frac{\alpha-1}{\alpha}\right)^{1/\alpha}/\sigma \sim 1$ minute duration level; most episodes in our data set have a duration less than 1 minute, so described reasonably well by the Weibull model.
Acceleration Model Formulation
(Hamdar, Treiver, Kesting and Mahmassani, 2008)
Model Background (Wallsten et al., 2005): Psychology Experiment

2 Decision Alternatives

- Fill the Balloon with more Air
- Stop

Cash in Existing Amount of Money

Balloon does not explode: Increase Reward

Balloon Explodes: Lose all Money

Decision depends on:
1- Value function: how decision makers evaluate alternatives
2- Estimated Probability of Balloon Explosion

Decision Maker

Outline  Introduction  Background  The Model: Tactical Stage  Operational Stage  Numerical Results  Conclusion
Behavioral Framework (1): Model Structure

- In the Car-Following Process, three behaviors are possible:
  1. Drivers accelerate
  2. Drivers decelerate
  3. Drivers keep the same speed

- Choosing between these three behaviors is choosing between a set of acceleration values between \( a_{\text{min}} \) and \( a_{\text{max}} \).

- Driver \( n \) sequentially evaluate the collision risk involved in choosing the acceleration \( a_n \) and the corresponding gain or loss:
  
  1. The risk is represented by a collision probability
  2. The gain and losses are evaluated using a “prospect theory” value function allocated to each acceleration value.
Behavioral Framework (2): Prospect Theory

Prospect theory allows choosing between different set of alternatives (acceleration values) presented for individual $n$ (driver) at time $t$: $C_n(t)$.

Each alternative has a measured “objective” utility $O(C_n(t))$.

A value function $U_{PT}[O(C_n(t))]$ is used to transform the “objective” utility into a perceived “subjective” value. The value function can represent:

1. Risk seeking versus aversion attitudes expressed by humans (drivers).
2. Asymmetry in weighing gains and losses.
3. The tendency to measure gains and losses not in terms of absolute values but with respect to a reference point.

A probability $P(C_n(t))$ is associated to each alternative.

A prospect function $\pi[P(C_n(t))]$ transforms the probability terms to prospects.

Based on the prospect of each alternative, the alternative with the highest $\pi[P(C_n(t))] \cdot U_{PT}[O(C_n(t))]$ is chosen.
Estimation of Collision Probability (1)

What is the future speed of my leader?

\[ \sigma(v_{n-1}) \]

\[ v_{n-1}(t) \]

pdf of \( v_{n-1}^{est}(t) = f(v | t) = \frac{1}{\sqrt{2\pi}\sigma(v_{n-1})} \exp \left[ -\frac{(v - v_{n-1}(t))^2}{2\sigma(v_{n-1})^2} \right] \]

\( v_{n-1}^{est}(t) \): estimated (subjective) future speed of lead vehicle \( n-1 \) as perceived by driver \( n \) over anticipated time span \( \tau_n \)

Follows normal distribution with standard deviation \( \sigma(v_{n-1}) \), and mean equal to actual velocity of the leader \( v_{n-1}(t) \).
Estimation of Collision Probability (2)

- The crash probability $p_n(t + \tau_n)$ is given by the probability that the gap ($s_n(t + \tau_n) = x_{n-1}(t + \tau_n) - x_n(t + \tau_n) - L_{n-1}$) at time $t + \tau_n$ is $\leq 0$.

- Given constant acceleration for the follower (driver in question) and constant velocity for the leader:

$$x_{n-1}(t + \tau_n) = x_{n-1}(t) + \tau_n v_{n-1}^{est}(t) \quad x_n(t + \tau_n) = x_n(t) + v_n(t)\tau_n + \frac{1}{2}a_n\tau_n^2$$

$$p_n(t + \tau_n) = P\left\{ v_{n-1}^{est}(t) < \frac{v_n(t)\tau_n + \frac{1}{2}a_n\tau_n^2 - s_n(t)}{\tau_n} \right\}$$

- Writing $v_{n-1}^{est}(t)$ in terms of standardized normal distribution, we get:

$$v_{n-1}^{est}(t) = v_{n-1}(t) + \sigma\left(v_{n-1}\right)Z$$

$$p_n(t + \tau_n) = P\left\{ \frac{\Delta v_n(t)\tau_n + \frac{1}{2}a_n\tau_n^2 - s_n(t)}{\sigma(v_{n-1})\tau_n} < Z \right\} = \Phi\left( \frac{\Delta v_n(t)\tau_n + \frac{1}{2}a_n\tau_n^2 - s_n(t)}{\sigma(v_{n-1})\tau_n} \right)$$

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Evaluation Process (1)

- The gain and losses are expressed here in term of gains and losses in speed from the previous acceleration instance i-1.

- If the gain and losses are expressed in terms of an abscissa $\Delta v = a_n \times \tau_n$, the value function $U_{PT}(a_n)$ is defined as follows:

$$U_{PT}(a_n) = \frac{\left(w^+ + (1 - w^-) \cdot \tanh\left(\frac{a_n}{a_0}\right) + 1\right)}{2} \cdot \left(\frac{a_n}{a_0}\right)^Y$$

  - Normalizing Parameter (1 m/s²)
  - Positive Parameters to be Estimated

- In a collision, the loss is assumed to be related to a seriousness term $k(v, \Delta v)$ weighted by $w_c$: when the seriousness of the driver increases, $k(v, \Delta v)$ increases, represents the sensitivity to the loss caused by an accident.

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The value function is:

\[ U(a_n) = (1 - p_{n,i}) U_{PT}(a_n) - p_{n,i} w_c k(v, \Delta v) + \varepsilon_n \]

Where

- \( U_{PT}(a_n) \) = PT value function
- \( p_{n,i} \) = probability of colliding with rear-end bumper of lead vehicle given that no collision took place in the \( i-1 \)th acceleration instance
- \( \varepsilon_n \) = driver specific error term assumed to have Weibull distribution

Using a continuous logit model, the stochastic car-following acceleration \( a_{n,\text{car-following}}(t) \) of vehicle \( n \) is retrieved from the following probability density function:

\[
f(a_n) = \begin{cases} 
\frac{\exp(\beta \times U(a_n))}{\int_{a_{\text{min}}}^{a_{\text{max}}} \exp(\beta \times U(a')) \, da'} & a_{\text{min}} \leq a_n \leq a_{\text{max}} \\
0 & \text{otherwise}
\end{cases}
\]

where \( \beta \) is a free parameter (\( \beta > 0 \)) that reflects the sensitivity of choice to the utility.
Acceleration Model: Preliminary Implementation
All \( N+1 \) drivers (\( n = 0, 1, \ldots N \)) are assumed to have identical parameters where \( s \) is the corresponding gap and \( \Delta v \) is the relative speed (positive when approaching).

The estimation uncertainty \( \sigma (v_l) = \alpha v_l \) of the velocity of the leader is proportional to the velocity itself, i.e., the relative error (variation coefficient) \( \alpha \) is constant.

The anticipation time horizon \( \tau \) is assumed to be the minimum between the time-to-collision \( \tau_{TTC} = (s / \Delta v) \) and some maximum value \( \tau_{\text{max}} \):

\[
\tau = \tau(s, \Delta v) = \begin{cases} 
\frac{s}{\Delta v} & \Delta v \geq \frac{s}{\tau_{\text{max}}} \\
\tau_{\text{max}} & \text{otherwise}
\end{cases}
\]
Asymptotic Expansion

- an asymptotic expansion of the acceleration probability distribution of this model will give:

\[ \dot{v} = a \approx \sim N(a^*, \sigma_a^2) \]

→ the distribution of accelerations is approximately given by a Gaussian distribution whose moments are:

\[ a^* = \text{arg}(\max(U(a))), \quad \sigma_a^2 = \frac{-1}{\beta U''(a^*)} \]

- \( U'(a) \) and \( U''(a) \) can be calculated analytically (given by derivatives of \( \Phi(z) \) that is a density of a Gaussian). The value \( a^* \) itself needs to be calculated numerically (Iterative Procedure with Newtonian Method).
Acceleration Model: Numerical Testing Results
### Initial Plots (1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum anticipation time horizon</td>
<td>$\tau_{\text{max}} = 5$ s</td>
</tr>
<tr>
<td>Velocity uncertainty variation coefficient</td>
<td>$\alpha = 0.1$</td>
</tr>
<tr>
<td>Logit uncertainty parameter (higher for smaller uncertainty)</td>
<td>$\beta = 3$</td>
</tr>
<tr>
<td>Accident weighing factor</td>
<td>$w_* = 4.0$</td>
</tr>
<tr>
<td>Exponents of the PT utility</td>
<td>$\gamma = 0.5$</td>
</tr>
<tr>
<td>Weighing factor for the negative PT utility</td>
<td>$w^- = 2$</td>
</tr>
<tr>
<td>Minimum acceleration</td>
<td>$a_{\text{min}} = -8$ m/s$^2$</td>
</tr>
<tr>
<td>Maximum acceleration</td>
<td>$a_{\text{max}} = 4$ m/s$^2$</td>
</tr>
<tr>
<td>Acceleration normalizing factor</td>
<td>$a_0 = 1$ m/s$^2$</td>
</tr>
</tbody>
</table>
Initial Plots (2)

- Several Relationships (Value Function versus Space Headway, Value Function versus Speed, Value Function versus Relative Speed) were tested.

- Remarkably, in stochastic equilibrium, approximate time headways of 1.5 seconds are kept constant in the car-following regime (mainly influenced by $\alpha \tau_{\text{max}}$).
Contour plots of the acceleration probability density (equation 2) as a function of $v$, $s$ and $\Delta v$ (top left, top right and bottom left). Contour for a situation with a standing vehicle or a red traffic light, $v = \Delta v$ for $s = 30$ m (bottom right).

1. **Decrease in “a” as “v” increases:**
   - returning to safety gap

2. **Increase in Variance away from “a = 0”:**
   - PT value function

3. **Harsh decrease in “a” attempting avoiding collision (higher than “$v^2/2s$” if $\Delta v > 10$):**
   - slowly to reach stopping headway

4. **If at rest and $s = 30$ m, attempt accelerate slowly to reach stopping headway**

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- Numerical Results
- Conclusion
Moving Traffic with “s = 20 m”:

all vehicles use the maximal deceleration values when

\[ 18(\text{m/s}) \leq v \leq 40(\text{m/s}) \quad \text{and} \quad 7(\text{m/s}) \leq \Delta v \leq 20(\text{m/s}) \]

The lowest variances are observed mostly for small “v” and when \( \Delta v \) decreases below zero: less disturbances is caused by accelerating (mostly related to the vehicle properties) then decelerating (mostly related to a drivers’ personality).
Acceleration Model: Calibration using Genetic Algorithm
The Genetic Algorithm Technique (1)

- Finding an optimal parameter set for such car-following model is a nonlinear optimization problem that needs to be solved numerically.

- Since the relative error is overestimated for congested conditions and the absolute error is overestimated for free-regime, a mixed error term is chosen as the objective function to be minimized:

\[
F_{\text{mix}} [v_{\text{sim}}] = \sqrt{\frac{1}{v_{\text{data}}} \left\langle \left( \frac{v_{\text{sim}} - v_{\text{data}}}{v_{\text{data}}} \right)^2 \right\rangle}
\]

Where \( \left\langle . \right\rangle \) refers to the temporal average of a time series of duration.
The Genetic Algorithm Technique (4)

- For Sample 1, 205 vehicles out of 2052 vehicles were retrieved. As for Sample 2, 184 vehicles out of 1836 vehicles were removed. The average mixed errors for Sample 1 and Sample 2 were 13.75% and 13.83% respectively.
The Genetic Algorithm Technique (5)

- Clear inter-driver heterogeneity is observed.
- Interestingly enough, all parameters have a Gaussian like distributions with a clear peak except for the exponent of the PT utility Gamma following an exponential-shaped distribution.

---

**Outline**
- Introduction
- Background
- **The Model**
  - Tactical Stage
  - **Operational Stage**
- Numerical Results
- Conclusion
GA: other distributions (6)
GA: other distributions (7)

Reaction Time

![Bar chart showing reaction time distribution with bins for 0.3 to 2.1]
GA: other distributions (8)
GA: other distributions (9)

![Beta: Logit Uncertainty Parameter](chart.png)
GA: some differences between parameters across samples

**Sample 1**

**Sample 2**

**Outlines:**
- Introduction
- Background
- The Model
- Tactical Stage
- Operational Stage
- Numerical Results
- Conclusion
Some Simulation Results
Simulation Framework

- In this simulation exercise, during free-flow conditions, the acceleration value is that necessary to reach a driver’s desired speed and limited by maximum and minimum possible acceleration rates (vehicle property).

- The desirability of changing lanes is dictated by the hazard-based model.

- There is only the possibility of choosing one target lane while making a lane-changing manoeuvre.

- The possibility of changing lanes is assessed using MOBIL model:
  - When changing lane, the deceleration of the new-follower should not exceed the deceleration calculated through the “operational stage model:
Fundamental Diagram for the Integrated Model; Also shown are virtual one minute detector data with an on-ramp placed at x = 10, 9 and 6 kilometers.
Conclusions
Concluding Comments

- Translate and adapt concepts from cognitive psychology (learning, anticipation, risk aversion) to modeling driver behavior.

- Formulate models that can be calibrated and implemented in microscopic traffic simulation environment.

- Incorporate crash-inducing (collision probability and accident weighing parameter) risk-taking (probabilistic choice under uncertainty) behavior into microscopic operational simulation framework.

- Framework in which the driving process is viewed as a continuous process governed by the experiences encountered during the duration of different episodes; can capture state dependence, impatience rates, reaction times, anisotropy, adjacent lane effects.

- Linking microscopic and macroscopic sides of traffic modeling through episode-based concepts.
Thank you!

Questions?
**Major car-following models: Wiedemann Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AXadd</td>
<td>1</td>
</tr>
<tr>
<td>AXmult</td>
<td>2</td>
</tr>
<tr>
<td>BXadd</td>
<td>3</td>
</tr>
<tr>
<td>BXmult</td>
<td>4</td>
</tr>
<tr>
<td>EXadd</td>
<td>5</td>
</tr>
<tr>
<td>EXmult</td>
<td>6</td>
</tr>
<tr>
<td>OPDVadd</td>
<td>7</td>
</tr>
<tr>
<td>OPDVmult</td>
<td>8</td>
</tr>
<tr>
<td>CX</td>
<td>9</td>
</tr>
<tr>
<td>BNullmult</td>
<td>10</td>
</tr>
<tr>
<td>BMAX</td>
<td>11</td>
</tr>
<tr>
<td>BMIN</td>
<td>12</td>
</tr>
<tr>
<td>(V_{DES})</td>
<td>13</td>
</tr>
<tr>
<td>(V_{MAX})</td>
<td>14</td>
</tr>
</tbody>
</table>

\[
B(I) = \frac{1}{2} \* AX  \\
FaktorV = V  \\
BMIN = -BMAX  \\
BMAX = BMAXmult * (V_{MAX} - V * FaktorV)  \\
BNULL = BNULLmult * (RND2(I) + NRND)  \\
\]

\[
(1 - 2.NRND)  \\
\]

\[
\left(1 - (RND1(I).AXmult) + (RND1(I)).\sqrt{V} \right)  \\
\]

\[
\left(\frac{DX - AX}{CX} \right)^2  \\
\]

\[
\left(RND1(I) + RND2(I)\right)  \\
\]

\[
AX + EX * BX  \\
\]

\[
V = SDV * EX^2  \\
\]

\[
OPDV = CLDV * (-OPDVadd - OPDVmult * NRND)  \\
\]
Major lane-changing models
Standard Fundamental Diagram

Flow-Density Data for a full “traffic-state cycle” from the German Freeway A9-South near Frankfurt on July 31, 2001 (Treiber and Helbing, 2007)
Loss aversion seen in the steeper slope with losses than with gains: Asymmetry in weighing gains versus losses

Diminishing sensitivity to increasing gains and losses

Evaluation of outcomes relative to a reference point, taken as the speed in previous acceleration instance

(Kahneman and Tversky, 1979)
it is not guaranteed that $a^*$ is unique (nonlinearities in $U_{PT}(a)$).
However, all investigations presented in this paper show that it is unique for the parameters chosen while assessing the model.
<table>
<thead>
<tr>
<th>Acceleration Models</th>
<th>Ability to Capture Congestion/Breakdowns</th>
<th>Ability to Model Accidents</th>
</tr>
</thead>
<tbody>
<tr>
<td>GHR (GM) (Gazis et al., 1959)</td>
<td>GHR models failed to capture traffic breakdowns by offering over-estimated freeway capacities: unrealistically high velocities when drivers are following each other closely.</td>
<td>The calibrated value of the sensitivity factor $\lambda$ or its structure $\lambda = \frac{C}{S^n}$ prohibited the creation of accidents. Relaxation: randomness added on the stimulus term (relative velocity)</td>
</tr>
<tr>
<td>Gipps (Gipps, 1981)</td>
<td>The model looses its realistic property in its deterministic properties at the limits: drivers’ interactions in dense traffic are not well captured.</td>
<td>The explicit introduction of a safety condition in the model formulation prohibited any accident creation:</td>
</tr>
<tr>
<td>CA (Nagel and Shreckenberg, 1992) (Krauss et al., 1996)</td>
<td>Congested clusters characterized by typical start-stop-waves are found in freeway traffic.</td>
<td>Due to a safety constraint explicitly imposed by the modelers, unrealistic decelerations are used forcing drivers to stop in the available gaps in front of them: $v_{des} = \min[v(t) + a_{max}, v_{max}, s_{gap}(t)]$, $v(t+1) = \max[0, v_{des} - \sigma \cdot n_{ran}, 0, 1]$, $x(t+1) = x(t) + v(t+1)$ Relaxation: $s_{gap}(t)$ is replaced by $s_{gap}(t) + 0.1$</td>
</tr>
<tr>
<td>Acceleration Models</td>
<td>Ability to Capture Congestion/Breakdowns</td>
<td>Ability to Model Accidents</td>
</tr>
<tr>
<td>---------------------</td>
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<td>---------------------------</td>
</tr>
<tr>
<td>SK (Krauss and Wagner, 1997)</td>
<td>The SK model is able to capture the metastable state but still cannot imitate the hysteresis effects and the traffic instabilities near-traffic breakdown.</td>
<td>SK Model is a modified CA model with a desired velocity concept adopted from the Gipps model. Accordingly, like Gipps’ safety condition, a safe velocity is imposed with: ( v_s \leq v_{safe} = b(\alpha_{safe} + \beta_{safe}) ) Relaxation: the above safe velocity term is increased by 0.27 m/s</td>
</tr>
</tbody>
</table>
| IDM/IDMM (Treiber et al., 2000) (Treiber and Helbing, 2006) | IDM and IDMM are some of the few models claiming that they are able to capture multiphase states in the fundamental diagram (congestion build up, stop and go waves, then traffic deterioration). | With an acceleration equation of: \[
\dot{v}_\alpha = a^{(\alpha)} \left[ 1 - \left( \frac{v_\alpha}{v_0^{(\alpha)}} \right)^{\delta} - \left( \frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \right)^2 \right]
\] An implicit car following mode is imposed with: \[
b_{int}(s_\alpha, v_\alpha, \Delta v_\alpha) = -a^{(\alpha)} \left( \frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \right)^2
\] \[
s^*(v, \Delta v) = s_0^{(\alpha)} + s_1^{(\alpha)} \sqrt{\frac{v}{v_0^{(\alpha)}}} + T^{(\alpha)} v + \frac{v \Delta v}{2a^{(\alpha)} b^{(\alpha)}}
\] This structure of the car following equation above (spacing in the denominator) and last term of the desired spacing prohibit the creation of accidents. Relaxation: removing the last term of the desired spacing |
| Wiedemann (Wiedemann, 1974) | Traffic Breakdown is captured in Wiedemann Model | An emergency braking mode will impose the vehicle to stop before an accident is generated: \[
B(I) = \frac{1}{2} \frac{DV^2}{AX - DX} + B(I - 1) + BMIN * \frac{ABX - DX}{BX}
\] Relaxation: replacement of the emergency braking mode with the normal braking mode |

**Appendix**