Comparative Analysis of Sequential and Simultaneous Choice Structures for Modeling Intra-Household Interactions

Peter Vovsha
PB Consult, Parsons Brinckerhoff, Inc
5 Penn Plaza, 17 Floor
New York, NY 10001
Phone: 212-613-8807
e-mail: Vovsha@pbworld.com

John Gliebe
Northwestern University and
PB Consult, Parsons Brinckerhoff, Inc.
5801 Osuna Road, N.E., Suite 200
Albuquerque, NM 87109
Phone: 505-878-6533, Fax: 505-881-7602
e-mail: gliebe@pbworld.com

Eric Petersen
PB Consult, Parsons Brinckerhoff, Inc
230 West Monroe St.
Chicago, IL 60606
Phone: 312-803-6513
e-mail: Petersene@pbworld.com

Frank Koppelman
Northwestern University
2145 Sheridan Road
Technological Institute A318
Evanston, IL 60208
Phone: 847-491-8794
e-mail: f-koppelman@northwestern.edu
Abstract
Intra-household interactions constitute an important aspect in modeling activity and travel-related decisions. Recognition of this importance has recently produced a growing body of research on various aspects of modeling intra-household interactions and group decision making mechanisms as well as first attempts to incorporate intra-household interactions in regional travel demand models.

The previously published research works were mostly focused on time allocation aspect and less on generation of activity episodes, trips, and travel tours that are necessary units for compatibility with regional travel demand models. Also, most of the approaches were limited to household heads only and did not consider explicitly the other household members as acting agents in the intra-household decision making. The current paper presents an attempt to build a general framework for incorporation of intra-household interactions in the regional travel demand model.

The proposed approach distinguishes between three principal levels of intra-household interactions: 1) Coordinated principal daily pattern types, 2) Episodic joint activity and travel, 3) Intra-household allocation of maintenance activities. The adopted models are discrete choice constructs of the Generalized Extreme Value class. The resulting model structures are discussed, with emphasis on the advantages of the simultaneous approach relative to the sequential method, as well as implications for practical applications.

It is shown that the choice structures can be reduced to a combination of a limited number of typical models like the parallel choice, participation choice, and partition choice models. These models together create an analytical framework for integrative modeling of the daily activity and travel of multiple household members, taking into account their interactions.
1. Introduction

Intra-household interactions constitute an important aspect in modeling activity and travel-related decisions. Recognition of this importance has recently produced a growing body of research on various aspects of modeling intra-household interactions and group decision making mechanisms as well as first attempts to incorporate intra-household interactions in regional travel demand models.

The previously published research works were mostly focused on time allocation aspect and less on generation of activity episodes, trips, and travel tours. In particular, the works of Borgers et al, 2002; Ettema et al, 2004; Fujii et al, 1999; Gliebe & Koppelman, 2002; Golob & MacNally, 1997; Goulias, 2002; Meka et al, 2002; Townsend, 1987; Zhang et al, 2002; Zhang et al 2004; and Zhang & Fujiwara, 2004 give examples of models for time allocation between various type of activities and household members. Though these works provide valuable insights into the intra-household decision-making mechanism they are not directly compatible with the structure of most travel demand models that are based on discrete units of travel and discrete choice modeling technique.

Most of the approaches including Borgers et al, 2002; Ettema et al, 2004; Gliebe & Koppelman, 2002, 2004; Golob & MacNally, 1997; Simma & Axhausen, 2001; Scott & Kanaroglou, 2002; Srinivasan & Bhat, 2004; Townsend, 1987, were limited to household heads only and did not consider explicitly the other household members as acting-active agents in the intra-household decision making. In most cases, the model structure was essentially built on the assumption of a “binary” household and could not be easily extended to incorporate more than two interacting agents. This is another limitation that has to be lifted in order to integrate intra-household interactions in the framework of regional travel demand models.

So far no comprehensive approach has been proposed that would address interactions between all household members, include all types of individual, joint, and allocated activities, and also represent an operational framework that could be incorporated in regional travel demand models. The current paper presents an attempt to build a general framework for incorporation of intra-household interactions in the regional travel demand model.

The proposed approach distinguishes between three principal levels of intra-household interactions:

- **Coordinated principal daily pattern types.** We consider three principal daily pattern types: (1) mandatory (work, university or school activities, which might include additional out-of-home non-mandatory activities); (2) non-mandatory travel (only non-mandatory activities at least one of which is out of home); and (3) staying at home or absence from town for the entire day. Statistical evidence shows strong coordination between household members at this principal level, resulting in such decisions as staying home for child care; coordinated work commutes; and household members taking time off together for major shopping trips, family events and vacations.
• **Episodic joint activity and travel.** Even if household members have chosen different pattern types (for example, one mandatory and the other non-mandatory) they may participate in shared activities and/or joint travel arrangements. We propose a classification of typical joint activity and travel types that support the development of operational choice models. In particular, we distinguish fully joint travel tours for shared activities from partially joint tours, in which household members share transportation without participation in the same activity.

• **Intra-household allocation of maintenance activities.** Many of the routine household maintenance activities (shopping, banking, visiting post office) are implemented individually; however, generation of such an activity and its allocation to a particular household member is a function of a household decision-making process. Thus, these activities require an intra-household interaction mechanism to be properly understood and modeled.

In view of these multiple layers of intra-household interaction, coordinated modeling of the individual daily activity patterns of several household members presents a complicated task, both conceptually and technically. Conceptually, two lines of integrity should be integrated: “vertical” integrity means that all activities and tours made by the same person should be modeled realistically, taking into account time-space constraints; whereas “horizontal” integrity means that decisions made at each level should be modeled jointly for the household members involved. These concepts give rise to complex analytical structures, presenting a challenge to practical modeling.

The resulting model structures are discussed, with emphasis on the advantages of the simultaneous approach relative to the sequential method, as well as implications for practical applications. The model structures considered in the paper are exclusively discrete choice models. Though some other alternative ways to model intra-household interaction suggest themselves (time allocation models, structural equations) and probably even have advantages over discrete choice structures in ability to capture various intra-household interactions in a holistic way, discrete choice approach better fits the micro-simulation framework adopted for most of the recent advanced regional travel demand models.

The paper is organized in the following way. The next two sections (2-3) present operational classification of intra-household interactions and ways of incorporation of intra-household interactions in a framework of regional travel demand model. Then the major section (4) presents a detailed description of choice models associated with intra-household interactions of each type. After that, in the Section 5, the resulting choice structures are discussed from the formal point of view. The last section (6) contains concluding remarks.


2. Classification of Intra-Household Interactions

The operational structure of intra-household interactions adopted for the current research distinguishes between two principal mechanisms – activity coordination and resource allocation – see Figure 1 below.

The activity coordination mechanism reflects the way how household members interact in order to undertake various joint activities and/or travel arrangements as well as allocate household maintenance tasks to household members. It is based on the general behavioral phenomenon that joint participation in activities has an added “group-wise worth” that cannot be reduced to a simple sum of individual utilities for each participant. It is also represents various compromises made by some household members in order to serve the other household members representing “altruistic” behavior that cannot be explained by the individual utility maximization. Activity coordination mechanism is in the focus of the current paper.

Resource allocation represents another facet of intra-household interactions. Even if activity agenda of all household members on a given day includes only “pure” individual activities, they have to interact in order to allocate constrained resources between them. In the context of travel demand modeling, the most important allocated (and frequently
constrained) resource is household cars. First attempts to incorporate intra-household allocation of cars as a part of a travel demand model have been made by Wen & Koppelman, 1999, 2000. This aspect of intra-household interactions is beyond the scope of the current paper. It represents an interesting and important topic for future research.

Activity coordination mechanism can be stratified by three following principal layers of intra-household interactions:

1. **Coordinated principal daily pattern types at the entire-day level.** We consider three principal daily pattern types: (1) mandatory (work, university or school activities, which might include additional out-of-home non-mandatory activities); (2) non-mandatory travel (only non-mandatory activities at least one of which is out of home); and (3) staying at home or absence from town for the entire day. Statistical evidence shows strong coordination between household members at this principal level, resulting in such decisions as staying home for child care; coordinated work commutes; and household members taking time off together for major shopping trips, family events and vacations.

2. **Episodic joint activity and travel.** Even if household members have chosen different pattern types (for example, one mandatory and the other non-mandatory) they may participate in shared activities and/or joint travel arrangements. We propose a classification of typical joint activity and travel types that support the development of operational choice models. In particular, we distinguish fully joint travel tours for shared activities from partially joint tours, in which household members share transportation without participation in the same activity.

3. **Intra-household allocation of maintenance activities.** Many of the routine household maintenance activities (shopping, banking, visiting post office, etc) are implemented and scheduled individually; however, generation of such an activity and its allocation to a particular household member is a function of a household decision-making process. Thus, these activities require an intra-household interaction mechanism to be properly understood and modeled. Maintenance task allocation mechanism may not be observed completely within a one-day framework since most of the maintenance tasks have cycles longer than one day.

It is also assumed that all else being equal, a general hierarchy of intra-household decision making follows these three layers from top to bottom. It means that entire-day level decisions come first. Then, conditional upon the chosen daily pattern types for each household member, the decisions regarding joint activities and travel are made. Finally, maintenance activities are allocated to persons conditional upon the chosen daily patterns and participation in joint activities. These assumptions give a schematic and simplified view on the extremely complicated real-world variety of travel behavior of the members of a household and numerous interactions between them. This view, however, has two important features:

- The proposed structure gives a good coverage for most frequent cases of intra-household interactions observed in the household travel surveys; also many complicated cases of joint activities and travel arrangement that do not fall
directly under one of the proposed categories, still should be only slightly simplified or split in order to be brought in line with the proposed structure.

- The propose structure serves as a constructive framework for derivation of operational choice models that can be estimated based on available household travel surveys and applied in a framework of regional travel demand models.

Further classification of episodic joint activities is subject to the purpose of travel demand modeling. At this stage we do not model explicitly in-home activities. Thus, only out-of-home activity episodes associated with travel are analyzed and classified. The following principal categories of episodic joint activity and travel are distinguished – see Figure 1 above:

1. **Joint travel generated by the shared activity.** This category is almost exclusively bound to non-mandatory activities (shopping, eating out, other maintenance, and discretionary activities) as well as almost exclusively implies a fully joint tour structure. Cases where fully joint tours include an escorting function (recorded drop-off or pick-up purpose for one of the household members) have been assigned to the escorting category. Cases where fully joint tours include mandatory activity (work, university, school) have been assigned to the category of ride-sharing for mandatory activity. In the last case, joint travel occurs as a result of time-space synchronization of individual activities and implies only a travel co-operation without sharing the activity. Thus, the modeling technique for synchronized mandatory activity cannot employ a joint destination choice and time-of-day scheduling as in the case of shared activity, but rather should link individually made choices in time and space. Rare cases where activities reported by different members of the travel party within the same fully-joint tour proved to be different (say shopping and discretionary) were considered as individual tours.

2. **Joint travel to synchronized mandatory activities.** This category has a significant share of drop-offs and pick-ups of school children made by workers on the way to and from work. Additionally, a significant percentage of school children travel together to and from school generating fully-joint tours and joint half-tours. Also carpooling of workers for commuting to work is observed, though this type has a comparatively low percentage. Enote that even going to the same school is not considered as shared activity because it finally has an individual character and joint travel lasts as long as there is a time-space synchronization of the individual activities.

3. **Escorting that is reported the purely “altruistic” purpose of driving some other household member without participation in the activity.** Statistical analysis has shown that majority of escorting is associated with serving children who cannot drive alone and, in the case of preschool children, cannot even ride transit alone.

The proposed classification gives rise to five choice models (daily activity pattern, joint activities, ride-sharing, escorting, and task allocation) that will be discussed below in detail along with possibilities to apply them in a simultaneous or sequential fashion as well as integrate them in the framework of a regional travel demand model.
3. Incorporation of Intra-Household Interactions in a Regional Travel Demand Model System

The ultimate purpose of the current research is a development of an operational system of models that can handle intra-household interactions in the framework of a regional demand model. This purpose dictates several necessary requirements to the models of intra-household interactions:

- Full coverage of all household, person, and activity types; from this perspective rather certain simplifications of the modeling structure would be acceptable, rather than exogenous segmentation out by some particular household types, person types, or activities.

- Compatibility with the tour-based model structures that are used for modeling destination and mode choices; in particular in many cases tours and half-tours (outbound and inbound) are used as units of decision making rather than activity episodes or durations (time allocations).

- Compatibility with the micro-simulation modeling paradigm that requires probability distributions to be associated with the ranges of modeled parameters; for this reason discrete choice structures are preferred to models that operate with continuous average values.

A general framework for incorporation of intra-household interactions in a travel demand model system is shown in the Figure 2 below.
Figure 2. Intra-household interactions in travel demand model system

A fully-fledged regional travel demand model system includes all types of joint, allocated, and individual activities in combination with the corresponding travel. From this point of view, the system of intra-household interactions described in the previous section and the five choice models resulting from the adopted assumptions constitute the upper-level activity generation set of models. They are followed by a set of models for non-mandatory individual activities (individual mandatory activities are modeled first as a part of a daily pattern choice). In this framework it is assumed that decisions made on individual non-mandatory activities are conditional upon the decisions made regarding mandatory and joint activities.

The micro-simulation framework allows for explicit modeling of individual households and persons as interacting agents. A constructive and operational approach to modeling intra-household interactions arises from modeling activity generation process at different levels – in some cases at the level of entire household and in some other ones at the level of persons. In particular, the following typical mechanism can be mentioned:

- Daily activity pattern type choice relates to persons and the household level serves only for coordination of individual choices made by each person.
- The decision-making unit for non-mandatory joint activities is assumed to be a household while person decisions relate to participation in joint activities.
- The interaction mechanism for ride-sharing for mandatory activities is different from the first two; these activities have already been generated and the choice to participate formally relates to the person; however linkage of travel tours of different persons occurs at the household level.
- The interaction mechanism for escorting children is also unique; demand for escorting comes from the individual activities (mandatory and non-mandatory) generated at the person level; then (possible) pooling of these activities together (if there are several demands at the same time) and allocation to the “chauffeurs” is implemented at the household level.
- Finally, maintenance tasks are generated by the entire-household needs and then allocated to persons.

The tour-based framework considers travel tours as units of modeling. Each travel tour has a primary destination associated with the primary activity and (possibly) additional stops on the way from the anchor location to the primary destination (outbound half-tour) and from the primary destination back to the anchor location (inbound half-tour). Location and associated arrival/departure times for each half-tour and intermediate stop are modeled by means of the tour/activity-level choice models that are not discussed in this paper. In the regional model system, these choice models are closely intertwined with the tour/activity generation models through time-space constraints. For the models, discussed in the current paper, we assume that the corresponding attributes (for example, residual time windows left for each person after scheduling mandatory activities, or location of mandatory activities considered for ride-sharing) are known and fixed. More
details about the regional model system framework as well as examples of incorporation of intra-household interactions can be found in Vovsha et al, 2003, 2004a, 2004b.
4. Choice Structures: Sequential and Simultaneous Approaches

Based on the general framework described in the previous sections we now consider operational formulations for all related choice dimensions. A principal model design relates to the way how various choice dimensions are processed – simultaneously or sequentially – and if sequential approach is applied it is important to define the sequence in the most behaviorally realistic way. Simultaneous approach in the ultimate version means formulation of one single complicated choice model that would include all types of activities and interactions across household members. Gliebe (2004) and Gliebe and Koppelman (2004) adopt this approach to represent the joint choice of joint, partially joint or distinct daily activity patterns (DAP). The difficulty with this approach is the large number of individual and joint DAP that must be included to successfully represent the range of alternatives available to two or more person households. Consideration of this range of alternatives may be infeasible for practical model application. Thus, the balance of this discussion considers that some decomposition of the choice structure is necessary. It is obviously an insurmountable task. Thus a certain decomposition of the choice structure is necessary.

When deciding how to organize the sequence of models, several rules should be considered. In particular, sequential processing by activity types and/or by persons in the household creates useful and operational lines for decomposition of the “grand” model. Decomposition by activity types assumes that a certain hierarchy of activities is imposed. For example, a hierarchy of activities follows from the layers of intra-household interactions in Figures 1 and 2 above already suggests a constructive decomposition that is used as a base for most of the choice structures discussed below. Decomposition by persons is based on a predetermined intra-household hierarchy of person types in decision-making. In both principles of decomposition, sequential representation of choices reflects on the casuality of decision-making rather than chronological processing of decision steps.

These two principles of decomposition can be combined in numerous ways. Some choice dimensions can be processed simultaneously; some other ones can be processed assuming hierarchically of activities; while some other dimensions can be better processed by individual persons. All else being equal, simultaneous modeling of several choice dimensions is considered theoretically superior since it results in integrated model that captures all corresponding trade-offs across all dimensions. However, sequential approach offers significant practical advantages in terms of simplicity of each model, limited number of alternatives, and focused control of the model properties at the estimation and application stages.

The ways to combine simultaneous and sequential approaches with a maximum practical worth and emerging modeling structures are discussed below. The sub-section that follows (4.1) describes some general rules for decomposition by persons based on intra-household priority of different person types for different household compositions. The following sub-sections (4.2-4.6) are devoted to the five main models of intra-household interactions.
4.1. Household Composition and General Priority Rules

In the current research and model segmentation we distinguish between the following eight main person types that are mutually exclusive and collectively exhaustive; percentage shown for each type is calculated as an average of the last two comprehensive household surveys undertaken by the Mid-Ohio Regional planning Commission – MORPC and Atlanta Regional Commission – ARC, that served as a basis for estimation of the most of the models discussed in the paper:

1. Preschool child of the age under 6 (40.8%);
2. School child of the pre-driving age (6-15) distinguished from school child of driving age (16-17) since access to car has a significant impact on activity/travel behavior – 10.1%
3. School child of driving age (16-17) – 3.2%
4. University students, including all adults (18 or older) who have reported their main occupation as full-time studying in university / college even if they work part time – 3.3%
5. Full-time worker, including adults who have reported their main occupation as full-time work (35 hours or more a week) – 40.8%.
6. Part-time worker, including adults who have reported their main occupation as work, but have less than 35 hours a week – 5.3%
7. Non-working adult including persons who have not reported any regular work or studying activity (homemakers or temporarily unemployed) and under 65 years old – 12.9%.
8. Retired person of 65 years or older – 7.5%.

Household size is always equal to the total number of persons across categories 1-8. Household composition is defined in terms of the presence and number of persons of particular types. Additional important attribute of household activity/travel behavior is a life-cycle. In the current research we used 3 following life-cycle categories defined by the average age of the household heads:

- Young household (under 35)
- Middle-age household (35-64)
- Old household (65 and older)

Table 1 below presents the results of the household cluster analysis across household size (number of adults), composition, and life-cycle dimensions. The results clearly indicated on eleven household types defined in terms of number of adults and household composition (adult type mix). Each household type is associated with a dominant majority of households of one particular life-cycle category. Also percentage of each household type and relative frequency of having children are shown.
Table 1. Typical household compositions

<table>
<thead>
<tr>
<th>Type</th>
<th>Life cycle</th>
<th>No of adults</th>
<th>Adult person type mix</th>
<th>Extensions with children</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>Young</td>
<td>1</td>
<td>University student</td>
<td></td>
<td>12.3%</td>
</tr>
<tr>
<td>Y2</td>
<td>Young</td>
<td>2</td>
<td>University student</td>
<td>U/W/N</td>
<td>33.0%</td>
</tr>
<tr>
<td>Y3</td>
<td>Young</td>
<td>3</td>
<td>2 University students</td>
<td>U/W/N</td>
<td>5.3%</td>
</tr>
<tr>
<td>Y4+</td>
<td>Young</td>
<td>4+</td>
<td>3 U or 3 W workers</td>
<td>U/W/N</td>
<td>30.7%</td>
</tr>
<tr>
<td>M1</td>
<td>Mid</td>
<td>1</td>
<td>Worker/Non-worker</td>
<td></td>
<td>15.4%</td>
</tr>
<tr>
<td>M2</td>
<td>Mid</td>
<td>2</td>
<td>2 Workers/Non-workers</td>
<td></td>
<td>43.6%</td>
</tr>
<tr>
<td>M3</td>
<td>Mid</td>
<td>3</td>
<td>2 Workers/Non-workers</td>
<td>W/N/U/R</td>
<td>34.3%</td>
</tr>
<tr>
<td>M4+</td>
<td>Mid</td>
<td>4+</td>
<td>3 Workers/Non-workers</td>
<td>R</td>
<td>33.3%</td>
</tr>
<tr>
<td>O1</td>
<td>Old</td>
<td>1</td>
<td>Retired</td>
<td></td>
<td>1.5%</td>
</tr>
<tr>
<td>O2</td>
<td>Old</td>
<td>2</td>
<td>Retired</td>
<td>R/W/N/U</td>
<td>3.0%</td>
</tr>
<tr>
<td>O3+</td>
<td>Old</td>
<td>3+</td>
<td>2 Retired</td>
<td>R/W/N/U</td>
<td>6.3%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Household type is an important aggregate discriminator for the household behavior in general and intra-household interactions in particular. It should be noted that only about 60% of households correspond to the traditional type with 2 adults (household heads). A significant percentage of households have either 1 adult or 3 and more adults and intra-household interaction mechanism in these households do not follow a standard framework of interactions between two household heads (male and female). In the estimation procedure, household type frequently serves for either full or at least partial segmentation of the model.

Each household composition type also associated with a certain intra-household hierarchy of person types. This hierarchy is important for sequential decomposition of complicated choice structures by persons. Persons of the higher priority in the corresponding choice context are assumed to make their decisions first, while persons of the lower priority make their choices conditional upon the choice made by the persons of the higher priority. The following statistical observation and subsequent assumptions have been made to justify the sequencing of person types adopted for the model system recently developed for MORPC:

- In general, adults adjust their schedules to serve children and not vice versa. For example, child sick at home with a very high probability will require from at least one of the adults to stay at home (for preschool children it was over 90% of cases for sickness and over 80% for other cases). Workers quite frequently take day-offs to take care on children. It is less usual when children do not go to school to take care on adults. Family vacations, though organized by adults, are also coordinated with school holidays when children are involved.

- Full-time workers and university students are characterized by the lowest percentage of joint daily activity patterns with children comparing to part-time workers and non-workers. Their schedules are less flexible than schedules of part-
time workers and non-workers. In this sense these person types are called more individualistic than part-time workers and non-workers in the paper. Thus, it makes sense to position university students and workers before part-time workers and non-workers.

- When we consider co-operation at the entire-day-level across household adults (having vacation together, going for major shopping jointly, etc) it is assumed that all else being equal it is easier for part-time workers and non-workers adjust their schedules to accommodate university students and full-time workers than vice versa.

- University students are considered as of relatively higher priority comparing to workers since they normally have less flexibility in their mandatory activities and also university students living in family households with workers are frequently grown-up children.

- Younger non-working homemakers are positioned after retired persons since the retired household members may be physically limited in implementing household maintenance task. In this sense, the younger non-workers normally take a role of the household member responsible for maintenance tasks.

- If there are several household members of the same type they are ordered by age in such a way that the youngest person is assigned a higher priority.

Application of these principles leads to the general order of processing person types shown in the Table 2 below.

**Table 2. General order of processing person types**

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</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>C</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y2</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y3</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y4+</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>M</td>
<td>C</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
</tr>
<tr>
<td>M2</td>
<td>M</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td></td>
<td></td>
<td>M</td>
</tr>
<tr>
<td>M3</td>
<td>M</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td></td>
<td></td>
<td>M</td>
</tr>
<tr>
<td>M4+</td>
<td>M</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td></td>
<td></td>
<td>M</td>
</tr>
<tr>
<td>O1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>O3+</td>
<td></td>
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<td></td>
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<td>A</td>
</tr>
</tbody>
</table>

A – preferable cluster for simultaneous modeling of adult household members

C – preferable cluster for simultaneous modeling of household children

M – preferable cluster for simultaneous modeling of mixed parties
When modeling household travel behavior, there is an intermediate level of simultaneity between two extremes – considering the entire-household and sequential processing by household members. This level refers to intra-household clusters of persons that have especially strong interactions and coordination in decision making and joint/coordinated activities. When a household is modeled the clusters can be processed sequentially while all persons in the cluster are modeled simultaneously. Since clusters have a simple, predetermined, and limited structure the corresponding simultaneous model may prove manageable while simultaneous model of the entire household can be problematic because of the wide variety of household compositions.

In particular, the following intra-household clusters prove to be the most important and effective for modeling:

- 2-person adult clusters (frequently referred as “household heads”) in the households types Y2, M2, and O2
- 3-person adult clusters (most frequently household heads with a grown-up child or older family member living in; however this composition may be different for non-family households):
  - All adults in the household types Y3 and M3
  - Three first adults by person priority in the Table 2 in the household types Y4+, M4+, and O3+.
- 2-person children cluster that includes two youngest of the preschool or pre-driving school children for the household types Y1, Y2, Y3, and Y4+.
- 2-person children cluster that includes two oldest of the pre-driving or driving school children for the household types M1, M2, M3, and M4+.
- 2-person mixed cluster that includes the youngest non-worker and the youngest pre-school child in the households M1, M2, M3, and M4+.

Depending on the model context order of processing clusters can be changed though it normally follows the general person priority rules in the Table 2. Household members that are not included into the clusters are each formally considered as a separate cluster and they processed sequentially conditional upon the core cluster model outcomes.

### 4.2. Coordination of Daily Pattern Types

Classification of daily activity patterns (DAP) can be done in many different ways. DAP definition normally includes a list of activities undertaken by the person in the course of entire day with some predetermined hierarchy of the activity types. DAP may also include activity sequencing / scheduling attributes, as well as travel related characteristics. In particular, the definition adopted for the MORPC model system and for most other tour-based model systems uses travel tours as basic units.

**Figure 3** below shows the structural dimensions along which DAPs are classified in the current structure.
Daily activity-travel pattern

Mandatory

Workday
- Work tours
- 1, 2+

University day
- University tours
- University & Work tours
- 1, 2+

School day
- School tours
- School & Work tours
- 1, 2+

Non-mandatory

At home / absent

Figure 3. Classification of Daily Activity Patterns (DAP)

DAP is classified by three main types:

- **Mandatory pattern** that includes at least one of the three mandatory activities – work, university, or school. This constitutes either a workday or a university/school day, and may include additional non-mandatory activities such as separate home-based tours or intermediate stops on the mandatory tours. Work at home for at least 2 hours is also considered as a workday pattern that can be combined with work on tour or non-mandatory tours.

- **Non-mandatory pattern** that includes only maintenance and discretionary tours. By virtue of the tour primary purpose definition, maintenance and discretionary tours cannot include travel for mandatory activities.

- **At-home pattern** that includes only in-home activities. At the current stage of model development, at-home patterns are not distinguished by any specific activity (work at home, take care of child, being sick, etc). It should be noted that for simplicity, cases with complete absence from town (business travel) were combined with this category.

Mandatory DAPs are further classified by purpose and frequency of mandatory tours. The nature of mandatory activities – they are usually associated with both long duration and long commuting times – limits significantly the number of mandatory tours that can be implemented in the course of a day. The vast majority of observed cases include only one or two tours, where two-tour combinations include either two tours to the same primary activity or a combination of work and university/school activities.

In contrast to the mandatory DAP type, the non-mandatory DAP type includes a wider variety of tour frequencies and purposes that is difficult to cover by one choice framework. Thus, the associated details are modeled later in the model stream.

Statistical analysis presented in Vovsha et al, 2004a has shown that there is an extremely strong correlation between DAP types of different household members, especially for
joint non-mandatory travel DAP and joint staying at home (or having vacations together). It means that joint staying at home or having a non-mandatory travel day has additional utility beyond a person utility associated with these patterns when implemented alone. For this reason, DAP for different household members cannot be modeled independently.

In the most general way the DAP type choice model can be represented by a matrix view in the Table 3 below. Each household member \( m \in M \) has a row while available alternatives are represented by columns. The choice is associated with a value of 1 in the corresponding cell. The row totals are all equal to 1, while the column totals are not controlled and can take any value between 0 and \( M \) assuming that every alternative is available to every person.

Table 3. Coordinated DAP model – a matrix view

<table>
<thead>
<tr>
<th>HH members (( m \in M ))</th>
<th>DAP alternatives (( i \in I \cup J ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Individual Mandatory (( i \in I ))</td>
</tr>
<tr>
<td></td>
<td>W1</td>
</tr>
<tr>
<td>1st person</td>
<td></td>
</tr>
<tr>
<td>2nd person</td>
<td></td>
</tr>
<tr>
<td>3rd person</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Alternative DAP types are broken into two groups. The first group \( i \in I \) contains patterns with mandatory activities that assumed individual in a sense that choice of this pattern by one household member does not directly affect choice of the mandatory pattern by the other household member. The second group \( i \in J \) contains two patterns – staying at home and having only non-mandatory travel – that have a potential to be joint if several household members choose the same pattern.

The total number of possible matrices is equal to \((I + J)^M\) and this gives a maximum choice set for a simultaneous model. However, there are several important considerations that significantly reduce a dimensionality of the simultaneous model. First of all, most of the individual mandatory DAP types are only partially available for the corresponding person types. Secondly, and even more importantly, intra-household coordination of DAP types is relevant only for the NM and H patterns. Thus, simultaneous modeling of DAP types for all household members is essential only for trinary choice (mandatory, NM, H) while sub-choice of the mandatory pattern can be modeled for each person separately. These considerations result in the following alternative choice constructs – see also the Figure 4 that gives the corresponding tree representations:

- Sequential processing of persons according to the intra-household hierarchy.
- Simultaneous modeling of potentially joint alternatives for all household members.
• Parallel choice structure that considers combinations of main trinary choices at the upper level and individual sub-choices of mandatory alternatives simultaneously in one choice structure.

• Sequential processing of persons in a random order with cycling relations between them.

Sequential processing of persons according to the intra-household hierarchy assumes that choices made by the persons modeled first are used as variables explaining choices of the subsequently modeled persons. Choice model for each person includes all individual and joint alternatives available for the person type. Linkage across person is implemented by means of using Boolean variables for potentially joint choices (indicators of either staying at home a having a non-mandatory travel day) in the utility functions for the corresponding alternatives of the subsequently modeled persons. This was the preferred model structure for several regional travel demand models in US (New York, MORPC, and ARC) mostly for its simplicity in estimation and application. However, this approach does not have a full integrity in capturing intra-household interactions and relies heavily on the ordering of persons in the household according to the Table 2 above.

Simultaneous modeling of potentially joint alternatives for all household members assumes that for each person only a trinary choice (mandatory, non-mandatory, and
staying at home) is considered. Sub-choice of the mandatory alternative is done by a separate choice model conditional upon the choice of mandatory alternative in the trinary choice. Compared to the sequential model, this structure is much more powerful for capturing intra-household interactions in the most integrative way. Even for a household of 6-six persons the simultaneous combination of trinary models results in a total of $3^6 = 729$ alternatives that is a manageable number in a choice model estimation and application. For a limited number of households of size greater than 6-six, the model can be applied for the first 6-six household members by priority while the rest of the household members can be processed sequentially conditional upon the choices made by the first six members. First experience with this model structure has shown that the most meaningful and also technically convenient way to specify this model is to include up to 3-three adult household members (the main adult cluster) and up to 3-three (youngest) children in the simultaneous estimation. Additionally, a full segmentation of the model by 11-eleven household types shown in the Table 1 proved to be useful since it reduced a size of the main adult cluster for many segments as well as propensities for sharing DAP types were quite different in different segments. However, this structure has also a drawback comparing to the sequential estimation. Higher integrity of intra-household interactions comes at the expense of disjoining the upper-level trinary choice (M, NM, H) from the lower-level choice of the mandatory DAP sub-alternative for each person. The log-sum from the lower-level choice can be pre-calculated and used as a variable in the upper-level choice, but the estimation of the lower-level model should be done independently from the upper-level model. From this point of view a parallel choice structure that can combine advantages of the sequential and simultaneous models looks as a promising avenue for future research.

Parallel choice structure considers combinations of main trinary choices at the upper level and individual sub-choices of mandatory alternatives simultaneously in one choice structure. Different from just mechanical combination of all alternatives for all household members that would result in infeasible choice structure, the parallel choice model requires only a limited principal combinations to be listed explicitly (like as in the simultaneous approach described above) as upper-level nests (Gliebe 2004; Gliebe and Koppelman 2004). These nests correspond to the combination of activities where joint participation is essential. The structure of these nests captures different levels of intra-household interaction. Under each nest, the correspondent individual choices of mandatory alternatives are considered for each person individually. This greatly reduces the dimensionality of the model and makes the whole structure manageable in estimation and application. An example of a simple parallel choice structure for a case of a two-person household is shown in the Figure 5. The parallel choice structure may result in a more complicated choice model than simple multinomial and nested logit models. In particular, the cross-nested logit formulation suggests itself. A formal description of the parallel choice model is done below in the section 5 on the core model formulations (section 5.1).
Sequential processing of persons in a random order with cycling relations between them is an empirical structure that can be used only in the micro-simulation framework. In this structure, similar to the sequential processing by priority, every person has a choice model that includes all DAP type alternatives and the persons are processed sequentially. There are however three important differences in this approach compared to the sequential processing by priority. First, persons are processed in a random order, thus prioritizing by person type and age is not needed. Secondly, every person type model includes indicators on non-mandatory and staying-at-home patterns of all other household members along the two lines shown in the Figure 4 (having a joint non-mandatory day or staying at home together). This means that for example, the structure of inter-relations has cycles. For example, the choice model for full-time worker can include an indicator of a school child at home, while the choice model for school child can include and indicator on a full-time worker staying at home. There is no principal difficulty to estimate a model with cycling relations based on the observed data. Thirdly, this model is applied for all household members several times i.e. the whole set of the household members is re-iterated several times in a random order. Before the first iteration is implemented, all intra-household indicators are set to zero. Then, after each individual choice the indicators are updated, thus, the subsequently modeled person choice will take into account the previously made choices. First experience with this structure has shown that implementing several iterations through all household members with a full set of interaction variables creates a realistic and unbiased picture for the household with a good replication of the observed entire-household and person patterns.
4.3. Episodic Joint Non-Mandatory Activities

Episodic joint non-mandatory activities are characterized by fully-joint travel tours and fully-shared participation in activity of all member of the travel party. So we assume further on that the unit of modeling is a full tour that actually can be subdivided into several sub-episodes / stops. From the travel demand point of view each fully joint tour is considered as a unit of modeling with a group-wise making decision regarding the primary destination, mode, frequency and location of stops etc. This makes this type of intra-household interactions principally different from the other type of episodic interactions where activity sharing is not assumed (ride-sharing for mandatory activities and escorting).

From the formal point of view, modeling joint activities involves two linked stages:

1. **Generation stage** attributed to the entire-household level that is done by means of a **frequency-choice model** that considers a number of joint tours $j_k = 0,1,2...J_k$ as alternatives where $k \in K$ denotes segmentation of tours by purpose / activity type.

2. **Participation stage** at which decision is made for each household member $m \in M$ and tour $j_k = 0,1,2...J_k$ whether to participate or not in the joint tour.

Taking into account that the number of travel purposes is limited to 4-5 (shopping, other maintenance, discretionary, eating-out, visiting relatives and friends, etc) and the observed maximum total number of fully joint tours implemented by a household during a regular workday is limited to 2-3 (i.e. $J_1 + J_2 + ... J_k + ... = J \leq 3$), a single simultaneous frequency-choice model can be formulated that would cover all possible frequencies and purpose combinations as alternatives. A frequency-choice model formulation adopted for the ARC model system included 5 purpose and maximum of 2 joint tours that resulted in 21 alternatives – see Figure 6.
The participation stage in the most general way can be viewed as a matrix where each household member has a row while joint tours are represented by column – see Table 4 below. Choice to participate is associated with a value of 1 in the corresponding cell.

**Table 4. Joint participation model – a matrix view**

<table>
<thead>
<tr>
<th>HH members (m ∈ M)</th>
<th>Joint tours (j ∈ J)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st tour (purpose k₁)</td>
</tr>
<tr>
<td>1st person</td>
<td>1</td>
</tr>
<tr>
<td>2nd person</td>
<td>1</td>
</tr>
<tr>
<td>3rd person</td>
<td>1</td>
</tr>
<tr>
<td>…</td>
<td></td>
</tr>
</tbody>
</table>

Different from the DAP type choice matrix in the Table 3 above, the row totals in the joint participation matrix are not controlled and can take any value from 0 to J since each person may choose not to participate in joint tours or participate in some of them, or participate in all of them. The column totals are constrained to be greater or equal to 2 since joint activity requires at least two members to participate. Household members having 1 in a particular column constitute the travel party for the tour that is a subset of household members m ∈ M_j ⊆ M.
Each person participation matrix constitutes a distinct alternative. Even if the number of tours is limited (say, 3) and number of household members is limited (say, 6) the resulting number of participation matrices would be \( \binom{2^2}{3} = \binom{2^2}{6} = \binom{6*5}{1*2} = 3,375 \). If the frequency choice is combined with the participation choice in one simultaneous structure, the total number of alternatives would be even greater, though not significantly since the matrices for frequencies of 1 and 2 would be much simpler. Choice structures with several thousands of alternatives are not infeasible if a parsimonious component-wise utility structure can be applied. However, for first practical implementations of the model, various decomposition schemes with sequential processing across some dimensions were applied. Overall, the following approaches to modeling joint activities can be outlined:

- **Sequential modeling of generation and participation stages**, that results in two choice models applied in succession:
  - Tour frequency choice model that is applied for the entire household and yields probability of having a certain set of joint tours by purpose
  - Person participation choice model that yields probability of having a certain participation matrix conditional upon the chosen set of joint tours; the participation choice model in itself quite complicated and can be decomposed by sequential processing of tours (i.e. columns of the Table 4). Further on, the participation model for each tour can be decomposed into two models – see also Vovsha et al, 2003 for more details regarding decomposition of the person participation model:
    - Party-composition choice model that predicts a principal party composition in terms of the participating person types (adults, children, mixed)
    - Person participation choice model that assigns member of the travel party for each tour conditional upon the chosen party composition; this model in turn can be decomposed by persons into a sequence of binary choice models (to participate or not to participate)
- **Simultaneous modeling of generation and participation stages** as one choice structure

Sequential modeling of generation and participation stages assumes that the tour-frequency choice model is applied first for the entire household. This choice model yields probability of a certain frequency combination of tours by purpose \( P(j_1, j_2, \ldots, j_K) \). The utility function is normally formulated in such a way that advantage can be taken on the combinatorial structure of choice alternatives each of them essentially represents a combination of elemental tours. In particular, the following utility structure was applied for the MORPC model:
\[
V(j_1, j_2, \ldots, j_K) = \alpha_{j_1+j_2+\ldots+j_K} + \sum_{k \in K} j_k \times V_k ,
\]

(1)

where:
\[
\alpha_{j_1+j_2+\ldots+j_K} = \text{constant that depends on the total number of tours only,}
\]
\[
V_k = \text{elemental utility of one tour specific to purpose.}
\]

In this structure it is assumed that the elemental purpose-specific utilities capture impact of all explanatory variables and if there are several tours in the alternative their utilities are linearly combined. Constants are specific to the total number of tours only (0, 1, 2…) and allows for capturing a saturation effect for multi-tour alternatives.

The participation choice model yields probability of a certain participation matrix to be chosen. The participation matrix can be described in terms of Boolean variables \( \{ \omega_{jm} = 0,1 \} \) or alternatively in terms of travel parties formed for each tour \( M_1 = \{ m \in M | \omega_{jm} = 1 \} \). If we consider tours sequentially, than the core choice model is essentially formulated for each tour separately and we can drop index \( j \) from the participation variables, i.e. consider only one column in the Table 4. Thus the participation problem is reduced to defining a probability \( P(\tilde{M}) \) of a subset of the household members \( \tilde{M} = \{ m \in M | \omega_m = 1 \} \) to be chosen as a travel party. Even this reduced task is not simple for the model formulation and estimation because of variety of person types and possible combinations of them. One of the constructive ways to decompose this choice further applied in the framework of the MORPC model system is to define first a principal party composition \( g \in G \) in terms of participating person types \( g \) and then model person participation conditional upon the chosen party composition – see Figure 6.

Possible party compositions \( g \in G \) are defined in a mutually exclusive and collectively exhaustive way by person types. For example, in the MORPC model, three party compositions were defined – adults, children, and mixed. Every household member \( m \) can participate in only a subset of relevant parties \( G_m \). There are in a common case several possible parties that can be formed within the same composition type \( \tilde{M} \subset M_g \). For example, is there are 3 adult household members there are 4 different adult parties that can be formed (1st and 2nd member, 1st and 3rd, 2nd and 3rd, and all three). The probability of a party to be chosen \( P(\tilde{M}) \) cane be written as:

\[
P(\tilde{M}) = P(g|G) \times P(\tilde{M}|M_g),
\]

(2)

where:
\[
P(g|G) = \text{marginal party composition choice probability,}
\]
While the first choice sub-model for marginal party-composition probability is comparatively simple and has a predetermined set of alternatives (three in the MORPC case), the second model for conditional participation choice probability is more complicated and the number of alternatives for this model is a function of the household size in composition. For example, consider a number of alternatives for adult travel party. If the household has only one adult member then adult party is infeasible. If the household has 2 adult members, adult party is feasible but participation of both adults is mandatory, i.e. there is only one participation alternative. If the household has 3 adult members, there are 4 possible ways to form an adult party. If the household has 4 adult members, there are already 11 possible participation alternatives. In this situation, it is not straightforward to formulate a choice structure that would incorporate all possible household sizes. One of the ways to simplify this model is to decompose it further into a sequence of binary choice model for each relevant person – see Figure 6. It means that the conditional participation probability is assumed to have the following form:

\[
P(M|g) = \prod_{m \in M_g} P(m|g),
\]

where \( P(m|g) \) is a probability for a person to participate in the given party composition.

The advantage of the binary participation choice model is its simplicity in terms of the number of alternatives. The choice utility can incorporate numerous person, household, and other variables as well as the purpose of the tour / activity type. Application of the binary choice model in a micro-simulation fashion does not guarantee a feasible travel party of size 2 or large. In the case of infeasible size of 0 or 1 the micro-simulation procedure is restarted until a feasible solution is generated. The model is automatically sensitive to the household size in a sense that relatively large parties would generally be generated for large households.

Though the application experience of these sequential structure (tour frequency, party composition by tours, person participation by persons) has shown that is performs reasonable well in practical terms. However, there are several serious weaknesses of the sequential approach that should be understood:

- Person participation in any joint tour is modeled independently of other tours; it is assumed that since number of tours is limited, there is only a little saturation effect. In reality, in a case that there several joint tours implemented by the household in the course of a day there can be strong limitations for participation of the same person in all of them including scheduling conflicts when these joint tours are implemented by different travel parties at the same time.

- Person participation in any joint tour is modeled independently of the other household members; it is assumed that every person has an inherent propensity to participate in joint activities of a certain type (purpose and party composition). In
In reality, there can be a strong clustering effect when participation of some household members may be strongly linked to participation of the other ones. Alternatively there can be a substitution effect, especially for mixed parties where one of the household adults takes children while the spouse may stay at home or undertake some other activity.

- Frequency of joint tours is modeled independently from person participation in them. Person participation is conditional upon the set of generated tours, however, there is no upward linkage that would make frequency of tour explicit function of potential person participation.

Consider the first two aspects that relate to the person participation under condition of a fixed set of tours. To ensure integrity across both tours and persons we have to consider the whole participation matrix and not to decompose it by columns (tours) or rows (persons). However it is not a simple task to find a choice model structure that would correspond to such multidimensional choice in terms of the observed utility components and correlations across unobserved components. A simple multinomial logit model that would consider all possible participation matrices with additive utility that is compound of components specific to either tour or person can be equivalently decomposed into a sequence of independent participation models. In particular, in the framework of logit-based models it is important to substantiate possible meaningful rules for grouping alternatives (participation matrices) based on some partial similarities.

One of the advantages of the matrix view in the Table 4 above is that it suggests several aggregations that can be effectively used for the model formulation. First of all, there are aggregations naturally associated with the marginal totals, i.e. total number of tours in which each person participates as well as total number of persons participating in each tour. Secondly, there are aggregations associated with clusters of persons and subsets of similar activities (tours for the same purpose). The following practical conclusions for substantiating the corresponding choice tree can be made:

- Nesting by persons with subsequent sub-nesting by a number of tours in which the person participates (person “workload”) can be useful since it gives a reasonable dimension for similarities across different participation matrices. Two matrices can be considered similar if all or at least some persons participate in approximately the same number of tours in both matrices. Alternatively, matrices that represent significantly different person workloads should be treated as distinct alternatives.

- Nesting by activities with subsequent sub-nesting by a number of participants (party size) also can be useful since it gives another dimension for similarities across different participation matrices. Two matrices can be considered similar if all or at least some tours have approximately the same number of participants. Alternatively, matrices that represent significantly different party sizes for all tours should be treated as distinct alternatives.

- Intra-household clusters by person types can be used as additional nests above persons or as alternative nests instead of persons or even along with the person
nests. This means that workload variations within the cluster are more probable, while changing workloads between clusters lead to principally different matrices. Travel party composition types used for the model decomposition in the MORPC project (see Figure 6 above) can be also associated with person clusters that do not necessarily have to be mutually exclusive (see Table 2 above).

- Tour groups by purpose can be used as additional nests above tours or as alternative nests instead of tours or even along with the tour nests. This means that participation variations between tours made for the same purpose are more probable, while changing parties between tours of different purposes lead to principally different matrices. This dimension, however, is relevant only for infrequent cases with 3 or more joint tours made by a household on the same day.

Combining these considerations we arrive at the following choice structure shown in the Figure 7 for a case of a 3-person household with 3 joint tours. Since there are 4 possible parties for each tour we have \(4^3 = 64\) possible participation matrices at the lowest choice level.

Clusters that correspond to persons that are substitutable, for example two adults each of them can play a role by a driver of a mixed party with children, are treated as nests (i.e. unobserved similar components in the utility function). Person clusters that correspond to person types that are most frequently linked in the same party (for example non-working adult with preschool child) are treated through observed components of the entire-party utility function.
Since this structure is not a simple hierarchy where each lower-level alternative belongs to exactly one nest and it cannot be modeled by a simple nested logit model. However, it can be effectively handled by generalized nested structures of the GEV class discussed in the section on choice structures below. A structure of the utility function can be quite parsimonious because it is essentially combined of a limited number of pre-determined components. This greatly simplifies the model estimation and application. In particular, the following components should be taken into account:

- \( V_{km} \) = elemental participation utility (person suitability for the activity)
- \( V_{mn} \) = person workload utility to capture saturation effects at the person level
- \( V_{kl} \) = party size utility to capture additional worth of joint participation

The participation matrix utility can be linearly combined of these components in the following way:

\[
W(P_{ij}) = \sum_{j} \sum_{m} V_{k(j),m} + \sum_{m} V_{m,n(m)} + \sum_{j} V_{k(j),l(j)}.
\]  

Each component corresponds to either the cell of the matrix or one of its margins (or aggregate margins) and they are all combined over (non-zero) cells.

The further generalization of the model includes the frequency choice aspect. Combining frequency choice and participation choice in one choice structure essentially means that the variety of participation matrix to consider should include all possible matrices with variable (rather than fixed) number of columns. There are several ways to construct such a simultaneous model:

- Consider frequency of joint tours as one more upper level (above the choice of participation matrix) in the choice hierarchy shown in the Figure 7. This is the most straightforward way to combine frequency and participation choice model since when they are considered in a sequential fashion, frequency should naturally precede participation. This, however, means that a similar structure should be replicated under each frequency combination. It is probably not the best way to construct a simultaneous model.

- Consider frequency as one more upper level only for tours, while person nest will continue to go to the root directly. This structure is simpler since additional nesting level and the corresponding multiplication of nests would relate to the tour side only.

- Consider a maximum observed number of tours for each purpose but allow for participation matrix to have empty columns with no persons assigned. This requires extension of the participation matrix rules (either no one or at least 2 persons have to participate) that may look artificial in the joint tour context. However, it looks reasonable and natural in a more general context of joint participation and individual allocation matrix that would be considered later in the section on choice model formulations.
All three approaches mentioned above produce the same full set of participation matrices with variable number of columns as elemental choice alternatives. The differences relate to the way how the nests are structured and consequently how the correlation structure of the model is assumed.

4.4. Ride-Sharing (Joint Travel Arrangements) for Mandatory Activities

Joint travel arrangements for mandatory activities (work and school/university) constitute a second major type of episodic intra-household interactions—see Figure 1 above. The principal difference between ride-sharing and joint activities discussed in the previous section is that ride-sharing does not assume joint activity and limited to joint travel arrangements only. If in the case of joint non-mandatory activities the whole joint tour is considered as one unit with joint decision-making regarding activity location and duration followed by joint travel arrangements, ride-sharing for mandatory activities relate to pure travel arrangement while the underlying activity for each participant is assumed individual with correspondingly individual choices of locations and durations. Thus, different from joint activities, the ride-sharing modeling technique does not require a generation model but rather a linking / synchronizing model.

Ride-sharing arrangements may require limited adjustments of schedules of participants in order to synchronize their travel but it is assumed that locations for each (mandatory) activity and the basic schedules are fixed and predetermined for each household member and the household interaction are aimed at finding the best travel arrangements that would serve that individual locations and schedules rather than change them. By virtue of ride-sharing it is bound to auto mode only. Theoretically speaking joint travel arrangements may include transit trips as well; however, explicit linking of joint transit trips does not bring significant modeling benefits. Also, the observed share of joint rides to mandatory activities by transit proved to be close to zero.

When modeling ride-sharing it is assumed that for each household member it is known a number and purpose of mandatory tours $I^k_m$ as well as location zone $z(i)$, preferred outbound time (departure from home) $\tau(i)$, and preferred inbound time (arrival back home) $\pi(i)$ for each tour.

Realistically, frequent ride-sharing can occur only for either outbound or inbound bunches of mandatory tours that share the same home end. Thus, the model can be essentially broken into two parts—outbound and inbound ride-sharing. In many cases, these two parts can be processed independently, especially when a worker and school child are involved with very different preferred activity durations. However, for worker-worker and child-child compositions, two-way ride-sharing arrangements can be considered where ride-sharing decisions are not independent by directions.

The ride-sharing model considers partition of mandatory tours into ordered subsets of outbound and inbound half-tours $j = \{i_1, i_2, \ldots\}$. The length of the subset corresponds to the
number of participants. One-tour length means travel alone; two-tour length means participation of two persons in a shared ride; three-tour length means participation of three persons in a shared ride, etc. Order of participants reflects their roles in the shared ride. The first tour corresponds to the driver; the second tour corresponds to the passenger with the longest ride (the last getting-off passenger for the outbound direction or the first getting-in passenger for the inbound direction), etc.

From the formal point of view modeling ride-sharing involves two subsequent stages:

1. Linkage and synchronization of outbound and inbound half-tours that is done by means of a partition-choice model that considers all possible partitions of mandatory half-tours into rides (alone and shared)

2. Ordered participation choice model that essentially considers a role of each participant (driver, passenger) and route along which activity locations of all ride participants are visited.

A tree choice structure example for a household with 2 workers (each having one work tour) and one child (having one school tour) is shown in the Figure 8 below.

The outbound and inbound sets of linking choices are not exclusive but rather should be combined. Thus, even for a comparatively small household of 3 persons, the linking-choice level includes $5 \times 5 = 25$ alternatives. For a large household the formal number of choice alternatives would grow up exponentially. However, in the real-world model
estimation and application there are numerous considerations that allow for reduction of the ride-sharing choice model to a manageable size:

- Number of household members who actually have mandatory tours (workers and students) is limited in most of the households to 3-4 persons with predominantly one tour per person. Thus, for majority of cases the choice structure would be similar to shown in the Figure 8 above

- Many of the possible linkages can be rejected as impossible at the preliminary stage of synchronizing locations and departure/arrival times based on reasonable thresholds. These thresholds include maximum allowable differences in departure/arrival times (30 min performs quite well in practical terms) and maximum deviation from the shortest path to or from the location of activity for the driver (5 miles performs quite well). Application of these thresholds allows for cutting down a number of branches in the linking model significantly. In fact, for majority of households it proved to be difficult to find more than one realistic linking alternative and most of them corresponded to the observed worker-child combinations

- A maximum size of travel party can be limited to 3 participants because larger travel parties for ride sharing to mandatory activities proved to be very infrequent even for large households.

In addition to a priori elimination of improbable alternatives, several constructive decompositions of the choice structure can be considered. The first one includes a natural breakdown into the linking and participation-role stages. Secondly, the linking stage itself can be implemented as a sequence of pair-wise choices rather than a single choice model. To illustrate the ways of decomposition of the linking model it is useful to put it into a matrix view – see Table 5 below.

**Table 5.** Ride-sharing model – a matrix view

<table>
<thead>
<tr>
<th>Drivers’ half tours</th>
<th>Passengers’ half-tours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outbound</td>
</tr>
<tr>
<td></td>
<td>1st worker</td>
</tr>
<tr>
<td><strong>Outbound:</strong></td>
<td></td>
</tr>
<tr>
<td>1st worker</td>
<td></td>
</tr>
<tr>
<td>2nd worker</td>
<td></td>
</tr>
<tr>
<td><strong>Inbound:</strong></td>
<td></td>
</tr>
<tr>
<td>1st worker</td>
<td></td>
</tr>
<tr>
<td>2nd worker</td>
<td></td>
</tr>
</tbody>
</table>

Each ride-sharing matrix constitutes a distinct alternative for the choice model. Rows correspond to half-tours of potential drivers. Columns correspond to all half-tours. Each column can have not more than one assignment of 1. A cell values of 1 means that the corresponding passenger (column) travels with the driver (row). Assignment of 1 to the diagonal cell means being a driver of the ride. The diagonal cell equal to 1 is mandatory.
for all rows that have non-zero totals (i.e. valid rides must include the driver’s half-tour). A column can consist of zeros only. It means that the corresponding passenger cannot find a ride and travel alone or by other modes (transit or non-motorized). A row can consist of zeros only. It means that the corresponding potential driver is either taken as a passenger by the other driver or travel alone or by other modes. A minimal positive total for each row is equal to two, meaning that it must be a driver and at least one passenger for each ride.

The table cannot have cycling relations when two half-tours serve as drivers to each other. Inbound and outbound half-tours cannot interact, thus the appropriate parts of the table are blocked out. Also, if the same person has several mandatory tours on the same day they cannot be linked between themselves. These rules along with the threshold described below introduce many a priori zeros to the ride-sharing matrix, thus, reducing significantly a number of feasible alternatives.

Every non diagonal cell is associated with potential pair of half-tours. A pair-wise measure of matching these half-tours in time and space can be formed as a linear combination of schedule discrepancies and route deviations associated with the ride:

\[ V_{i_1i_2} = a \times [\pi(i_1) - \pi(i_2)]^2 + b \times \left[ \min(L_{z_2} + L_{z_2z_1} - L_{z_1}, L_{z_1} + L_{z_2z_1} - L_{z_2}) \right]^2, \]  

(5)

where the coefficients \( a \) and \( b \) are statistically estimated together with the other choice model parameters. The minimum route deviation reflects the inconvenience of ride-sharing for the driver who has to visit the passenger’s location first and then go to his/her own location (in the outbound case). If one of the persons cannot drive (like in a case of worker-child carpool) the corresponding route deviation term is set to a large number, thus ensuring that minimum would relate to the driver’s deviation. Squaring discrepancies proved to work better than linear inclusion because the negative impact of large schedule discrepancies and route deviations on probability of ride-sharing is highly non-linear. In particular, a certain threshold on maximum allowable discrepancy \( V \) can be established based on the observed data.

This pair-wise measure is instrumental for both the model decomposition and simultaneous formulations. One of the possible model decompositions is based on sequential processing of pairs ordered by the minimum discrepancy measure. It assumes either independent processing of outbound and inbound directions or conditioning of the inbound direction upon the chosen outbound ride-sharing combinations. The rational behind a certain linking between outbound and inbound ride sharing is that the passenger for the outbound ride can be dependent on the ride back service since he/she does not have a car and transit service may not be available. Thus, tolerance to schedule discrepancies and route deviations may be generally higher for a two-way ride compared to a one-way ride.

Sequential processing of potential ride-sharing pairs requires development of a binary choice model that yields probability of sharing ride for two persons as a function of the matching measure as well as household, person, and other characteristics. For the
inbound direction the utility function of ride-sharing should include indicator on outbound ride sharing with generally a strong positive impact. Then the following sequential procedure can be outlined (outbound as an example):

1. Order outbound half-tour pairs by the matching measure (and possibly person type considerations)
2. Take the highest-order unprocessed pair that has a matching measure within the threshold $V_{ij} < F$
3. Run a binary choice model to calculate probability of ride-sharing and simulate the choice outcome (1-share, 0-not)
4. If the choice outcome is 1 than engage the pair:
   a. If no one of the engaged persons has been previously engaged yet, then register a new shared ride
   b. If one of the engaged persons has been already engaged in a shared ride, than add a new participant to the previously registered shared ride and mark all person pairs in this ride as processed
   c. If both engaged persons have been already engaged in different shared rides then combine these rides, register a new ride that includes all participants from the previous rides, discard the previous rides, mark all person pairs in a new combined ride as processed
5. If there are unprocessed pairs go to step 2; if no unprocessed pairs left then end

This procedure assumes a micro-simulation framework for the model application. Ordering of half-tour pairs by the matching measure can be enhanced using intra-household clusters in the Table 1 or any other person-based priority rules. For example, processing of worker-child pairs can be done prior to processing worker-worker pairs.

The simultaneous choice approach that relates to the upper-level nests in the Figure 8 is also practically manageable taking into account that numerous branches of the tree can be eat down pruned a priori. However, in some cases for large households the number of alternatives (i.e. ride-sharing matrices) can reach thousands. As for the most cases where choice alternatives are compound of elements, the utility function can be effectively combined from a limited number of components that greatly simplifies the choice model estimation and application even with thousands of alternatives. The following utility components can be used to construct the ride-sharing matrix utility:

$V_{ij}^{out}, V_{i}^{inb}$ = pair-wise half-tour ride-sharing utilities

$V_{ij}^{out}, V_{n}^{inb}$ = half-tour utilities specific to the party size of the ride

$V^{both}$ = utility of having a ride in both directions

The ride-sharing matrix utility takes the following form:
The person participation role model is a choice model that considers sequences of person within the ride in such a way that the first person plays the driver role, the second person corresponds to the passenger with the longest route, and so forth. The last person is the first passenger dropped off on the outbound half-tour or the last person picked up on the inbound half-tour. The last person does not experience any route deviation. The order of persons from the driver to the shortest-leg passenger corresponds to the magnitude of potential deviations from the shortest route. The number of alternatives is equal to \( n! \) and also some of them should be excluded since not every person can be a driver. For example, for a rare case of a ride-sharing of 4 persons and also assuming that they all can be drivers, we have \( 4!=24 \) alternative orders that all can be treated simultaneously in one choice structure. The utility for the participation role distribution is combined of person utilities associated with route deviation and additional driver-role component that is associated with person type and characteristics in the following way (example for a 3-person ride):

\[
V_{i_{1}i_{2}i_{3}} = V_{i_{1}}^{\text{dev}} + V_{i_{1}i_{2}}^{\text{dev}} + V_{i_{2}i_{3}}^{\text{dev}}. \tag{7}
\]

### 4.5. Escorting Children

Escorting is a joint travel arrangement that is characterized by distinctive-in-kind roles of participants. There is always an escorting adult driver (in vast majority of the observed cases a single adult person, otherwise it becomes a joint tour for shared activities) and one or several escorted children. Important characteristic that distinguishes escorting from all other joint activity and travel arrangements is that only the escorted persons have a purposed activity to participate while the driver does not participate in any activity and implement a pure chauffeuring function. A dominant share of escorting involves children as passengers. Escorting of adult household members is observed rarely and mostly in households with low car ownership. Thus, we assume escorting children from now on in this paper.

From the perspective of escorted person, he/she has a mandatory or non-mandatory tour to implement. The escorting service may cover the whole tour (two-way escorting) or only one of the half-tours (one-way escorting). From the perspective of the chauffeur, his/her tour may cover only one half-tour of the escorted child with no waiting at the activity location or both half-tours of the escorted child with waiting while the child is involved in his/her activity. Thus, for each tour of a child that demands escorting there are five possible alternatives:

- No escort
- Escort in outbound direction only (from home to activity)
- Escort in inbound direction only (from activity back home)
• Escort in both direction by means of two separate tours of the same driver or by different drivers without waiting
• Escort in both direction by means of a single tour of the same driver with waiting

To formalize escorting as a choice model we use the following notation:

\[ i \in I \] = child tours that demand escorting,
\[ m \in M_a \] = household adults that can play a chauffeur role,
\[ j \in J_m \] = escorting tours made by each chauffeur.

The set of children’s tours \( i \in I \) with all pertinent characteristics of the person \( m(i) \), tour purpose/activity type \( k(i) \), departure-from-home time \( \tau(i) \) for outbound half-tour, arrival-back-home time \( \pi(i) \) for inbound half-tour, and location \( z(i) \) is assumed known and fixed. The set of adult chauffeurs \( m \in M_a \) with all pertinent characteristics of the person and availability to serve child tours \( I_m^{out}, I_m^{inb} \) within the time window left after scheduling mandatory and joint activities (they are considered of higher scheduling priority) is also assumed known and fixed.

Modeling of escorting can be formalized as finding a set of escorting tours for each chauffeur \( j \in J_m \) where each tour covers a subset of outbound \( I_m^{out} \) and inbound \( I_m^{inb} \) children’s tours. The subsets \( I_m^{out} \) and \( I_m^{inb} \) are mutually exclusive across escorting tours \( j \in J_m \) for either outbound or inbound subsets. Also not every child half-tour that demand escorting can be satisfied in a general case. Thus, \( \bigcup I_m^{out} \subseteq I \) as well as \( \bigcup I_m^{inb} \subseteq I \). The escorting tour construction problem can be presented in the following matrix view – see the Table 6 below.

<table>
<thead>
<tr>
<th>Chauffeurs ((m \in M_a))</th>
<th>Escort tours ((j \in J_m))</th>
<th>Child tours demanding escort (i \in I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st child</td>
<td>2nd child</td>
</tr>
<tr>
<td></td>
<td>1st tour</td>
<td>2nd tour</td>
</tr>
<tr>
<td>1st chauffeur</td>
<td>1st escort (i)</td>
<td>(i)</td>
</tr>
<tr>
<td></td>
<td>2nd escort (i)</td>
<td>(i)</td>
</tr>
<tr>
<td>2nd chauffeur</td>
<td>1st escort (i)</td>
<td>(i)</td>
</tr>
</tbody>
</table>

Each escorting tour construction matrix constitutes a distinct alternative. The rules for construction of a feasible matrix are as follows:

- Every row corresponds to escorting tour of the chauffeur. Cell values of 1 correspond to served child half-tours. Escorting tours for each chauffeur are listed

<table>
<thead>
<tr>
<th>Chauffeurs ((m \in M_a))</th>
<th>Escort tours ((j \in J_m))</th>
<th>Child tours demanding escort (i \in I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st child</td>
<td>2nd child</td>
</tr>
<tr>
<td></td>
<td>1st tour</td>
<td>2nd tour</td>
</tr>
<tr>
<td>1st chauffeur</td>
<td>1st escort (i)</td>
<td>(i)</td>
</tr>
<tr>
<td></td>
<td>2nd escort (i)</td>
<td>(i)</td>
</tr>
<tr>
<td>2nd chauffeur</td>
<td>1st escort (i)</td>
<td>(i)</td>
</tr>
</tbody>
</table>
in a chronological order. The first escort tour can take any outbound or inbound child half-tours that fall into the available time window of the chauffeur $I^\text{out}_m, I^\text{inb}_m$. Each subsequent escorting tour of the same chauffeur has a narrower window available since the previous tour(s) block out additional time windows making available sets essentially tour-specific and dependent on the escorting matrix $I^\text{out}_{jm}, I^\text{inb}_{jm}$. The minimal row total allowed is 1 (at least one half-tour should be served; otherwise the escorting tour does not make sense). The maximum row total is controlled by the availability rules but it never can be greater than number of child tours in the set $\bar{I}$ multiplied by 2.

- Every column corresponds to a child half-tour (either outbound or inbound). The column total can be either 1 (get escorting) or 0 (not).
- The allocation of 1’s in each row should meet the internal tour feasibility conditions:
  - The bunch of outbound half tours of children $I^\text{out}_j$ served by the tour should have close departure-from-home times and locations. A threshold on the discrepancy measure described for ride-sharing above proved to be useful for bunching outbound half-tours for escorting as well.
  - The bunch of inbound half tours of children $I^\text{inb}_j$ served by the tour should have close arrival-back-home times and locations. The same threshold on the discrepancy measure is applied for bunching inbound tours as well.
  - All outbound half tours $I^\text{out}_j$ start earlier than inbound half-tours $I^\text{inb}_j$ served by the same escorting tour. Thus, only disjoint-in-time sets $I^\text{out}_j$ and $I^\text{inb}_j$ can be “bridged” by one escorting tour.

One of the problems with forming alternative escorting matrices and understanding the choice structure is that the set of escorting tours $J_j \in J$ itself is variable and should be considered as a part of the choice model. One of the possible views on the forming alternative escorting matrices that also gives some insights into the possible decomposition of the choice structure is that it can be broken into two subsequent stages:

1. Escorting tour set formation $j \in J$ by partitioning sets of children’s half-tours $I^\text{out}_j, I^\text{inb}_j$ into non-overlapping and non-exhaustive subsets $I^\text{out}_j, I^\text{inb}_j$. This problem is similar to the partitioning problem ride-sharing mechanism described above.

2. Escorting tour allocation to chauffeurs $j \in J_m$. This problem is similar to the task allocation mechanism described below.

However, this breakdown is not perfectly instrumental since the escorting tour formation cannot be done effectively without considering chauffeurs’ availability constraints. More exactly, if “bunching” outbound and inbound child half-tours can be effectively done
based on the half-tours themselves, “bridging” outbound and inbound half-tours cannot be done without imposing the chauffeur availability constraints.

Still, if we consider mechanically all possible tour formations and then allocation to chauffeurs, for most households, the task of listing all resulting combinations is not insurmountable. For example, in a household with 2 potential chauffeurs and 2 child tours (i.e. 4 half-tours) to handle, we will have 52 possible tour formation sets (including various options of serving only some of the 4 half-tours) and then from 2 to 16 allocation-to-chauffeurs alternatives for each of tour formation sets which results in approximately 500 escorting matrix alternatives. The corresponding choice tree is depicted in the Figure 9 below.

![Figure 9. Escorting choice tree – general case](image)

However, after application of the tour-feasibility checks many of the tour-formation sets fail at ether bunching or bridging stage. Further on at the chauffer availability check many of the matrices fail because at least one of the tours prove to be outside the available time window of the assigned chauffer. These two checks normally reduce the choice set size significantly. However, for large households the number of potential escorting matrices in the original list can come to thousands and thus decompositions of the choice structure is welcome even for computational efficiency.
One of the possible decompositions assumes that the household chauffeurs are ordered based on the ordering rules described in the Table 2 above but applied in the reversed order starting from the least individual person categories (i.e. non-workers are considered the first-choice chauffeurs, followed by retirees, then by part-time workers, then by full-time workers, then by university students, and finally by driving-age school children). Then, the choice model is developed for a single person and includes only residual chauffeuring alternatives left after the choices actually made by the previously modeled chauffeurs – see Figure 10 below as an example of a choice structure for 1 child tour to serve.

![Figure 10. Escorting choice tree for a single chauffeur](image)

Another possible decomposition is based on the ordering of child tours demanding escort rather than chauffeurs – see Figure 11 below for an example of 2 chauffeurs available. In this case, household children are ordered first by age (from youngest to oldest) and then tours for each child are ordered chronologically. The choice model considers one child tour at a time to define probability of coverage by escorting for outbound and inbound half-tours and assignment of the chauffeur. Bunching of child half-tours is done by inclusion of the chauffeur’s previously assigned tours as the lower-level sub-choices.
In all cases of the choice model formulation (either simultaneous or sequential by chauffeurs or sequential by child tours) only a limited number of components of the utility function should be considered which greatly simplifies the model estimation and application. The following most important component should be mentioned:

\[ V_{i}^{\text{out}}, V_{i}^{\text{inb}} = \text{escorting utility for each child half tour (no escort has zero utility)}, \]

\[ \Delta V_{i} = \text{additional child utility of escorting in both directions}, \]

\[ V_{im}^{\text{out}}, V_{im}^{\text{inb}} = \text{chauffeur suitability and availability for each child half-tour}, \]

\[ V_{mn}(T_{m}) = \text{chauffeur workload saturation effect}, \]

\[ V_{jm}(I_{jm}^{\text{out}}, I_{jm}^{\text{inb}}) = \text{chauffeur tour disutility associated with bunching and bridging}. \]

The entire-household escorting matrix utility can be expressed as a sum of these components in the following way:

\[ U = \sum_{i \in I} \left( V_{i}^{\text{out}} + V_{i}^{\text{inb}} + \Delta V_{i}^{\text{both}} \right) + \sum_{m \in M} \left( \sum_{i \in I_{m}^{\text{out}}} V_{im}^{\text{out}} + \sum_{i \in I_{m}^{\text{inb}}} V_{im}^{\text{inb}} + V_{m,n(m)} + \sum_{j} V_{jm} \right). \]  

(8)
The first two components correspond to different aspects of the utility of the served children while the last three components relate to associated disutility aspects of the chauffeurs.

4.6. Allocation of Maintenance Activities to the Household Members

Allocation of maintenance activities to household members represents another important aspect of intra-household interactions. Maintenance activities of this group and associated travel are implemented individually. In many cases a person implemented these activities has a certain freedom in scheduling them and also in organizing them by travel tours. For example, a worker can link shopping activity to the work tour as a stop on the way home from work or implement an additional home-based shopping tour after arrival home. These details relate to the person individual activity-travel pattern.

However, the need for shopping and other maintenance activity relate to the entire household rather than person who implements them. The essence of the current model is to generate these activities at the household level and then allocate to the household members. Since, travel details are dependent on the characteristic of the person and his/her individual decisions this model is better formulated in terms of maintenance tasks rather than tours. Maintenance task is defined as a sequence of chronologically adjacent episodes (stops) for the same purposes made either within mandatory tours or as independent tours. In most observed cases, there was not more than one maintenance task per mandatory tour (work, school, or university) as well as most of the home-based maintenance tours represented a single task.

From the formal point of view, modeling allocated activities involves two linked stages:

- Generation stage attributed to the entire-household level that is done by means of a frequency choice model that considers a number of maintenance tasks as alternatives where \(k \in K\) denotes segmentation of task by purpose / activity type.

- Task allocation stage at which decision is made for each task \(j_k = 1,2,...,J_k\) to which of the household members \(m \in M\) this task is assigned for implementation.

There is an appealing analogy between the maintenance task allocation problem and the joint non-mandatory activity problem described above. The frequency choice model has the same structure. The task allocation model has replaced the joint participation model. However, the task allocation choice model formulation is similar to the joint participation model with the only principal difference that a single person is assigned to each task instead of a party of several persons. This analogy will be further exploited in the model structure analysis.

Taking into account that the number of allocated maintenance activity types is limited to 2-3 (major shopping, grocery and incidental shopping, banking, other maintenance) and the observed maximum number of maintenance tasks implemented by a household on a regular workday is limited to 3-4, a single simultaneous frequency choice model can be formulated that covers all possible frequencies and activity types as alternatives. This
structure is similar to the frequency choice model for joint non-mandatory activities described above.

The allocation stage in a general way can be viewed as a matrix where each household member has a row while maintenance tasks are represented by columns – see Table 7 below. Allocation choice is associated with a value of 1 in the corresponding cell.

**Table 7. Task allocation model – a matrix view**

<table>
<thead>
<tr>
<th>HH members \ (m ∈ M )</th>
<th>Maintenance tasks \ (j ∈ J )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1(^{st}) task (purpose ( k_1 ))</td>
</tr>
<tr>
<td>1(^{st}) person</td>
<td>1</td>
</tr>
<tr>
<td>2(^{nd}) person</td>
<td>1</td>
</tr>
<tr>
<td>3(^{rd}) person</td>
<td></td>
</tr>
<tr>
<td>…</td>
<td></td>
</tr>
</tbody>
</table>

Similar to the joint participation choice matrix in the Table 4, the row totals are not controlled and can take any value from 0 to \(J\) since each person may not to take maintenance tasks, or take some of them, or take all of them. The row totals corresponds to person workloads. Different from the joint participation choice matrix, the column totals are constrained to be equal to 1 since each task is allocated to a single person. Preschool children are not considered as potential persons to allocate maintenance tasks. However all other person types shown in the Table 2 above including school children are considered generally available in the allocation model (if they have not chosen the stay-at-home DAP at the earlier modeling stage).

Each task allocation matrix constitutes a distinct alternative. Total number of possible task allocation matrices is \(M^J\) if we assume that every person can be assigned to any task. It should be noted that for a realistic situation of say a household with 4 members and 5 tasks, it will result in only 625 possible matrices that is quite manageable in a simultaneous choice structure, taking also into account that a parsimonious component-wise utility structure can be applied. However, for larger households where the number of allocation matrices can exceed 10,000, decomposition of the choice structure should be applied.

For example, in the current model system for MORPC the task allocation matrix is modeled by a sequence of two modeling steps (preceded by a frequency choice model) – see also Vovsha et al, 2004b for more details:

1. Task allocation choice model that is applied for each task independently and returns a choice of a person the most suitable for the task as a function of the activity type and person characteristics (person type, residual time window left after mandatory activities, the number of joint and escorting tours in which the person participates, etc).
2. Discretizing of the resulting fractional matrix of allocation probabilities $P(m|j)$ obtained at the previous stage. The entire matrix discretizing is applied with fixed margins $P_m = \sum_j P(m|j)$ and $1 = \sum_m P(m|j)$ that corresponds consequently to rows and columns of the matrix. The reason why matrix discretizing is applied instead of a simple Monte-Carlo pick for each row is that independent pick for each task may result in illogical allocations with one person overloaded while the others will not have a task. Thus, the matrix discretizing technique alleviates deficiency of the each allocation choice being modeled independently at the step 1.

The same way as we considered simultaneous formulations for the joint non-mandatory activity and escorting, it can be done for the task allocation model. Overall, the following approaches to modeling task allocation can be outlined:

1. **Sequential modeling of generation and allocation stages**, that results in two choice models applied in succession:
   - Task frequency choice model that is applied for the entire household and yields probability of having a certain set of allocation task by purpose
   - Task allocation choice model that yields probability of having a certain task allocation matrix conditional upon the chosen set of maintenance task to be implemented for the household; the task allocation choice model can be further decomposed by sequential processing of tasks or persons (columns or rows of the Table 7 above)

2. **Simultaneous modeling of generation and allocation stages** as one choice structure

The simultaneous structure is more complicated analytically that any of the particular models applied in sequence, but it has an important advantage because all choices are modeled in a fully consistent way. For example the sequence of models applied for MORPC had only a “downward” conditionality in a sense that each subsequent model was applied conditional upon the outcome of the previous model. However, there was no “upward” conditionality or variables like lower-level log-sums that would ensure that the upper-level choices made first do not contradict to the lower-level choice scopes. As the result, in the MORPC model system, task generation and allocation model used many duplicative variables. For example, the frequency choice model used a variable “maximum time window available across adults” while the allocation model used a variable “available window” for each adult.

Similar to the way how a simultaneous structure was introduced for the joint non-mandatory activity model above, consider first the task allocation matrix under condition of a fixed set of tasks. To ensure integrity both across tasks and persons we have to consider the whole allocation matrix and do not decompose it either by columns (tasks) or rows (persons). This choice model should properly take into account complicated structure of differential similarities across various matrices. The matrix view in the Table...
7 above suggests several aggregations that can be effectively used for the choice structure construction. Similar to the joint participation matrix, there are aggregations associated with the row marginal totals (person workloads in terms of the total number of tasks allocated) as well as clusters of persons and subsets of similar tasks (of the same purpose or activity type). However, there is no meaningful aggregation dimension associated with the party size since participation is limited to 1 person in for all tasks. The following practical conclusions for substantiating the corresponding choice tree can be made:

- Nesting by persons with subsequent sub-nesting by a number of tasks taken by the person can be useful since it gives a reasonable dimension for similarities across different task allocation matrices. Two matrices can be considered similar if all or at least some persons take approximately the same number of tasks in both matrices. Alternatively, matrices that represent significantly different person workloads should be treated as distinct alternatives.

- Intra-household clusters by person types can be used as additional nests above persons. This means that workload variations within the cluster are more probable, while changing workloads between clusters lead to principally different matrices.

Considering these nesting principles we arrive at the following structure shown in the Figure 12 for a case of a 3-person household (without preschool children) with 3 maintenance tasks generated. Since there are 3 person choices for each task we have $3^3 = 27$ possible allocation matrices at the lower choice level. The structure is obviously similar to the joint participation structure shown in the Figure 7 above with the major difference of absence of the party-size related nests.

![General choice structure for task allocation](image)

**Figure 12.** General choice structure for task allocation
Clusters corresponds to persons that are substitutable and have similar daily activity patterns, for example, two workers while the third person can be a school child with a different daily pattern and propensity to implement maintenance tasks.

Similar to the joint participation model, this structure is not a simple hierarchy and it cannot be modeled by a simple nested logit model. It requires generalized nested structures of the GEV class discussed below. A structure of the utility function can be again effectively combined of a limited number of pre-determined components. In particular, the following components should be taken into account:

\[ V_{km} = \text{elemental allocation utility (person suitability for the activity)} \]
\[ V_{mn} = \text{person workload disutility to capture saturation effects at the person level} \]

The participation matrix utility can be linearly combined of these components in the following way:

\[ U(m_j \in J, j \in J) = \sum_{j} V_{k(j),m} + \sum_{m} V_{m,n(m)} \cdot \]

The further generalization of the model includes the frequency choice aspect. Combining frequency choice and allocation choice in one choice structure essentially means that the variety of participation matrix to consider should include all possible matrices with variable (rather than fixed) number of columns. There are several ways to construct such a simultaneous model:

- Consider frequency of joint tours as one more upper level (above the choice of participation matrix) in the choice hierarchy shown in the Figure 12. This is the most straightforward way to combine frequency and allocation choices since when they are considered in a sequential fashion, frequency should naturally precede participation.
- Consider a maximum observed number of tours for each person nest but allow for the allocation matrix to have empty columns with no persons assigned. This requires extension of the allocation matrix rules (either no one or 1 person can be assigned to each task) that will be considered later in the section on choice model formulations.

Both approaches mentioned above produce the same full set of task allocation matrices with variable number of columns as elemental choice alternatives. The differences relate to the way how the nests are structured and consequently how the correlation structure of the model is assumed.
5. Typical Choice Model Structures

Most of the choice models described above result in a complicated choice structure when formulated as simultaneous choices. The complexity stems from the essentially multi-dimensional type of choice sets with differential similarities across different dimensions which cannot be incorporated by simple multinomial or nested logit models. Various decomposition rules described above allow for the model reduction to a sequence of simpler choice models each of them can be a multinomial or nested logit form. However, since the simultaneous choice structure has a clear advantage of integrity, it is important to explore more complicated model structures that can serve this purpose.

It is important to note that though there is almost an infinite variety of model formulations with respect to technical details, there is only a limited number of typical model structures that serve as core constructs for the model building. All models structure discussed below belong to the Generalized Extreme Value (GEV) class of logit-based models introduced by McFadden. An excellent explanation of the model structure, rules for constructing GEV models, and software available for the model estimation can be found in Bierlaire, 2002.

The following three typical choice structures are considered below:

1. **Parallel choice model.** This structure is useful for models like daily activity pattern choice of several household members and other choice situation where joint activities are considered along with individual activities and consideration of joint activities is important only across particular choice dimensions while we can assume independence of individual decisions across other dimensions. The parallel choice approach was developed by Gliebe (2004) and Gliebe and Koppelman (2004) to model the individual daily activity patterns of household heads, subject to their higher-level choice of joint interaction. This structure ensures that individual choices are consistent with and contribute to the utility of higher level joint choices and can include differential weights on the utility of each individual.

2. **Participation choice model.** This structure is useful for modeling participation in joint activities as well as individual allocation of tasks to the household members. This structure ensures that activities are generated at the household level and then participation choices are made in a consistent way across all activities and persons through intra-household, joint decision making.

3. **Partition choice model.** This structure is useful for modeling ride-sharing and escorting since it is designed to consider alternative pools of activities from a predetermined set. The model is designed to capture intra-household relative preferences in pooling different activities together as well as capturing compromises between convenience for passengers who obtain a ride and chauffeurs who experience additional “workload”.

The parallel choice, participation choice, and partition choice models together create an analytical framework for integrative modeling of the daily activity and travel of multiple
household members, taking into account their interactions. All models can be flexibly combined with each other as well as conventional choice models. In particular, modeling escorting services within a household can be effectively done by a combination of the partition choice model (to form escort tours) and participation model (to allocate escort tours to the chauffeurs). Each of the models is described below in detail.

5.1. Parallel Choice Structure

The proposed parallel choice model offers a way to constructively handle the situation where multi-dimensional choices are made in parallel by two or more by several household members with linkages across some of the choice alternatives. The advantage of the parallel choice model is that it treats joint decision making explicitly only a limited manageable subsets of combinations of alternatives chosen by different household members, with mutually exclusive subsets defined along a single choice dimension, while the rest of alternatives are considered to be chosen by the household members independently from each other. It greatly reduces the dimensionality of the resulting choice construct in estimation and application.

To better understand the parallel choice structure, we consider an example of two household members \( m_1 \) and \( m_2 \) making choices out of the choice sets \( i_1 \in I_{m_1} \) and \( i_2 \in I_{m_2} \) consequently. As an example of choices made in parallel can be choice of a daily activity pattern shown in the Figure 3 above. We are interested in predicting a two-dimensional choice of the pair \( i_1, i_2 \) out of the two-dimensional set of alternatives \( I_{m_1} \times I_{m_2} \). Let’s consider first two extreme cases for the model formulation – fully independent choices and fully joint choices.

Fully independent choices are characterized by the choice probabilities \( P(i_1) \) and \( P(i_2) \) that are independent of each other. Thus, the probability of the two-dimensional alternative would be equal to the product of probabilities for each person \( P(i_1, i_2) = P(i_1) \times P(i_2) \). If each individual probability is modeled by a multinomial logit model with utilities \( V_{i_1} \) and \( V_{i_2} \) consequently, the two-dimensional choice can be modeled by a single multinomial logit model with the utility function equal to the sum of individual utilities \( V_{i_1, i_2} = V_{i_1} + V_{i_2} \). However, in terms of a practical estimation and application of the model it is easier to calculate individual probabilities \( P(i_1) \) and \( P(i_2) \) and then combined them by the formula \( P(i_1, i_2) = P(i_1) \times P(i_2) \) rather than handle explicitly the two-dimensional choice.

Fully joint choices are characterized by choice probabilities that are significantly dependent on each other. Thus, the probability of a two dimensional alternative \( P(i_1, i_2) \) cannot be represented as a product of independent probabilities. Also, the utility function \( V_{i_1, i_2} \) cannot be broken into two independent parts \( V_{i_1} \) and \( V_{i_2} \). If one of the persons, say \( m_1 \) has a higher priority and makes his/her decisions independently we can assume that
the probability \( P(i_1) \) is independent on the choices made by the second person \( m_2 \), while the choices of the second person are conditional upon the choices made by the first person \( P(i_2|j_1) \). In this case the probability of the two-dimensional alternative would be equal to the product of probabilities \( P(i_1,i_2) = P(i_1) \times P(i_2|j_1) \) giving rise to the nested logit structure where the first person choices form nests while the second person choices form elemental alternatives under each nest. However, if this choice hierarchy cannot be assumed, we just have to consider two-dimensional joint choice of \( i_1,i_2 \). In both cases (nested logit with clear hierarchy and two-dimensional joint choice with unclear hierarchy) the utility function will have a significant interaction term \( \Delta V_{i_1,i_2} \) along with (possible) person-specific terms, i.e. \( V_{i_1,i_2} = V_{i_1} + V_{i_2} + \Delta V_{i_1,i_2} \).

In many real-life situations we are dealing with intermediate cases where choices of several household members are only partially dependent while the majority of other choices are independent – see the Figure 4 above that illustrates this for the example of daily activity pattern choice. In these cases the simple decomposition technique of fully independent choices cannot be applied. However, switching to a fully-joint structure is not a good solution either since it may result in huge dimensionality. The parallel choice structure takes an advantage of the partial interdependence of choice probabilities. It treats explicitly only these joint choices that have a significant impact on each other, while the majority of other choices are handled independently. It greatly simplifies the model structure and dimensionality in both estimation and application of the model.

The parallel choice structure is built on the assumption that the person choice sets \( i_1 \in I_{m_1} \) and \( i_2 \in I_{m_2} \) can be broken into mutually exclusive subsets \( j_1 \in J_{m_1} \) and \( j_2 \in J_{m_2} \) in such a way that the choices of alternatives \( i_1 \in I_{j_1,m_1} \) and \( i_2 \in I_{j_2,m_2} \) within the subsets can be considered individual. In this case, two persons choose a pair of subsets \( j_1,j_2 \) jointly while the subsequent choice of alternatives within each subset is done independently by each person. This structure is effective if the number of subsets is small enough to handle all possible pairs (triples, quadruples, etc if more than two persons are considered). The number of individual alternatives within each subset is not restrictive since they are modeled independently for each person and do not multiply the real dimensionality of the model.

In the example of daily activity pattern choice, each person has three subsets of alternatives – see the Figure 4 above:

1. subset of nine patterns with different combinations of mandatory activities
2. subset that includes only one alternative – non-mandatory travel day
3. subset that includes only one alternative – staying at home

This specification of subsets was based on reasonable assumption that mandatory activities are mostly individual and entire-day-level coordination between household members predominantly occur by having either non-mandatory travel day or staying at home (or absent for vacations) together. In general, when defining subsets for a parallel
choice model it is convenient to single out activities where joint participation brings significant additional utility.

Having 3 subsets for a two-person case results in a $3 \times 3 = 9$ upper-level nests in the parallel choice structure shown in the Figure 5 above. The lower-level choice is relevant only for patterns with mandatory activities and is modeled independently for each person.

The simplest GEV-class models that corresponds to the parallel choice assumptions has the following form that is essentially a nested logit model:

$$
G\left(Y_{ij2} = Y_{i1} \times Y_{i2}\right) = \sum_{j_1,j_2} \left[ \sum_{i_1 \in I_{j_1}} \sum_{i_2 \in I_{j_2}} Y_{ij2} \right]^\mu = \sum_{j_1,j_2} \left[ \left( \sum_{i_1 \in I_{j_1}} Y_{i1} \right) \left( \sum_{i_2 \in I_{j_2}} Y_{i2} \right) \right]^\mu.
$$

(9)

Where:

- $Y_{ij2} = Y_{i1} \times Y_{i2}$ = two-dimensional choice variables assumed to allow decomposition
- $0 < \mu \leq 1$ = nesting coefficient assumed to be in the unit interval

This form results in a simple nested choice structure where the nests correspond to pairs $j_1,j_2$ while elemental alternatives correspond to pairs $i_1,i_2$ unambiguously broken by nests. The choice utilities are assumed separable $V_{ij2} = V_{i1} + V_{i2}$ and additional worth of joint participation in the same activity is captured exclusively through nesting. Also, this form does not allow for an explicit assigning of differential importance weights to persons in group decision making at the nest choice level.

To account for interaction term in the utility function but preserving independence of individual choices within each nest we assume that this term is specific to the nest, thus the utility expression can be written as $V_{ij2} = V_{i1} + V_{i2} + \Delta V_{j_1(i_1)j_2(i_2)}$. The nest-level interaction terms $\Delta V_{j_1j_2}$ are supposed to be set to zero for subset pairs that do not include joint activities. In the case of daily activity pattern shown in the Figure 5 above only pairs that correspond to joint non-mandatory travel day and joint staying at home should have positive interaction terms. The GEV model structure that can incorporate this term is still a nested logit model of the following form:

$$
G\left(Y_{ij2} = Y_{i1} \times Y_{i2} \times \left(Y_{j_1(i_1)j_2(i_2)}\right)^\mu\right) = \sum_{j_1,j_2} \left[ \sum_{i_1 \in I_{j_1}} \sum_{i_2 \in I_{j_2}} Y_{ij2} \right]^\mu = \sum_{j_1,j_2} Y_{j_1j_2} \times \left[ \left( \sum_{i_1 \in I_{j_1}} Y_{i1} \right) \left( \sum_{i_2 \in I_{j_2}} Y_{i2} \right) \right]^\mu.
$$

(10)

To account for differential importance by persons we can introduce differential nesting coefficients by persons in the following way:
\[
G \left( \left( Y_{i_{j_{1}}} = Y_{i_{j}} \times (Y)_{i_{j_{1}}}^{v} \times (Y)_{i_{j_{2}}}^{v} \right) = \sum_{j_{1}, j_{2}} \left[ \sum_{i_{j_{1}, j_{1}}} \left( \sum_{i_{j_{2}}, j_{2}} \right) \right]^{v} = \sum_{j_{1}, j_{2}} \left[ \sum_{i_{j_{1}}, j_{1}} \left( \sum_{i_{j_{2}}, j_{2}} \right) \right]^{v} \right)
\]

(11)

where:
\[0 < v \leq 1\] = additional lower-level nesting coefficient.

This choice structure is a nested logit model with two nesting levels. The upper-level nests relate to joint activity subsets and have their own specific portion in the utility function (interaction term). The lower-level nests correspond to the choices made by the first person who is considered more important decision maker. The lowest level of elemental alternatives corresponds to the second person. Since the utility function is assumed separable by persons within each nest, the choices made by each individual conditional upon the chosen upper-level nest are still independent. However, the choice of the nest \( j_{1}, j_{2} \) would be more dependent on the attractiveness of alternatives \( i_{j_{1}, j_{1}} \) for the first person compared to attractiveness of alternatives \( i_{j_{2}, j_{2}} \) for the second person. In particular, assuming that individual portions of utilities \( V_{i_{j}} \) and \( V_{i_{j}} \) are of the same order of magnitude and \( v \) tends to zero, the second person choices would not affect the upper-level choice of the nest. The last would be almost exclusively defined by the first person. In the model estimation this coefficient can be made differential by nests \( v_{i_{j}, j_{2}} \) to capture differential priorities of different person types in different joint decision-making contexts.

### 5.2. Participation / Allocation Matrix Choice Structure

As was mentioned before in the sections on joint participation (in non-mandatory activities) and (maintenance) task allocation these two problems have a similar structure and can be considered as particular cases of the same general problem. The general problem can be stated as finding probabilities of participation matrices \( \omega \in \Omega \) of household members \( m \in M \) in activities \( j \in J \) where each participation matrix corresponds to a set of person participations (parties) by activities \( \{ M_{j}^{\omega} \}_{j \in J} \). For the joint participation case, person participations are limited to include at least two persons. Thus a choice set for joint participation matrices can be expressed in the following way:

\[
\Omega_{2+} = \{ \omega \in \Omega \mid M_{j}^{\omega} \in M_{2+}, \forall j \in J \},
\]

(12)

where:

\( M_{2+} \subset M \) = subsets of household members with 2 or more persons.

Similarly, a choice set for task allocation matrices can be expressed in the following way:

\[
\Omega_{1} = \{ \omega \in \Omega \mid M_{j}^{\omega} \in M_{1}, \forall j \in J \},
\]

(13)
where:

\[ M_1 \subset M = \text{subsets of household members with 1 person exactly.} \]

Subsets with 1 person exactly correspond to just household members \( m \in M \) themselves. Thus the expression for the task allocation set of matrices can be equivalently rewritten in the following form:

\[ \Omega_i = \{ \omega \in \Omega \mid M_j^\omega = m(j), \forall j \in J \}. \] (14)

The choice model structure for a general participation case (including both joint participation and individual allocation) can be derived from the following GEV function:

\[
G(\{Y_{\omega}\}) = \alpha \sum_{m \in M} \left[ \sum_{n=0}^{\text{\#}(\omega)} \frac{n}{N_{\omega}} Y_{\omega} \right]^\mu + \beta \sum_{j=1}^{\#(J)} \left[ \sum_{n=0}^{\text{\#}(j)} \frac{l}{N_{\omega}} Y_{\omega} \right]^\nu
\] (15)

where:

- \( \overline{J} \) = total number of activities in the set \( J \)
- \( \overline{M} \) = total number of persons in the set \( M \)
- \( N_{\omega} \) = total number of person-activity participations in the matrix \( \omega \)
- \( \omega \in \Omega_{mn} \) = matrices for which the person \( m \) participates in \( n \) activities
- \( \omega \in \Omega_{jl} \) = matrices for which the activity \( j \) has \( l \) participants
- \( \alpha, \beta \geq 0 \) = upper-level inclusion parameters normalized such as \( \alpha + \beta = 1 \)
- \( 0 < \mu, \nu \leq 1 \) = nesting coefficients assumed to be in the unit interval

Persons considered in the set \( M \) include only active household members on the given day (not staying at home). For individual task allocation it also excludes preschool children as well as excludes joint participation matrices with only preschool children assigned to one of the activities. We assume that these infeasible cases are already excluded from the set \( \omega \in \Omega \).

The upper-level inclusion parameters \( \alpha \) and \( \beta \) regulates importance of the 2 upper-level nests – person-based and activity-based. The large is \( \alpha \) the more choice of the participation matrix depends on the person workload factors and less depends on the party-size considerations. The lower-level inclusion parameters distribute matrices by nests associated with person workloads and party size for each activity. Each matrix is included into the person workload nests based on the proportion of each person workload in the total number of all person participations in all activities in the matrix \( n(m)/N_{\omega} \); the denominator is also equal to the total of all workloads across all household members. Each matrix is included into the party-size nest based on the proportion of the party size in the total number of all person participations in all activities in the matrix \( l(j)/N_{\omega} \); the
The denominator is also equal to the in the total number of all party sizes across all activities. Since the lower-level inclusion parameters are fixed by the matrix structure, only upper-level inclusion parameters \( \alpha \) and \( \beta \) have to be estimated along with the utility coefficients and nesting coefficients.

Consider now two particular cases described above – joint participation choice and individual task allocation choice. The joint participation choice model shown in the Figure 7 above can be obtained from the general expression for the participation GEV function by imposing the following constraints:

- Only matrices with party size equal or greater than \( 2 \), \( \omega \in \Omega_{2+} \), are considered
- Only party-size nests with 2 or mode participants \( l \geq 2 \) are considered

With these constraints the joint-participation GEV function can be written in the following form:

\[
G(J_{\omega}) = \alpha \sum_{j \in J} \left[ \sum_{n=0}^{7} \left( \sum_{\omega \in (\Omega_{\omega}, \Omega_{\omega})} \frac{n}{Y_{\omega}} Y_{\omega} \right)^{\mu} \right]^\nu + \beta \sum_{j \in J} \left[ \sum_{l=2}^{7} \left( \sum_{\omega \in (\Omega_{\omega}, \Omega_{\omega})} \frac{l}{N_{\omega}} Y_{\omega} \right)^{\mu} \right]^\nu. \tag{16}
\]

The individual task allocation model shown in the Figure 12 above can be obtained from the general expression by imposing the following constraints:

- Only matrices with party size equal to 1, \( \omega \in \Omega_{1} \), are considered
- Party-size nests can be eliminated since all parties are equal to 1, for this reason the upper level inclusion parameters can be eliminated (\( \alpha \equiv 1, \beta \equiv 0 \))
- The total number of participations is equal to the number of activities for all matrices \( N_{\omega} = \bar{J} \)

With these constraints and simplifications the individual task-allocation GEV function can be written in the following form:

\[
G(J_{\omega}) = \sum_{j \in J} \left[ \sum_{n=0}^{7} \left( \frac{n}{\bar{J}} \sum_{\omega \in (\Omega_{\omega}, \Omega_{\omega})} Y_{\omega} \right)^{\mu} \right]^\nu. \tag{17}
\]

Now consider generalization of the participation matrix choice to the case where the number of activities is unknown, i.e. combine frequency and participation choices. One of the most effective ways to implement this generalization is to consider the set of activities \( J^* \) as a maximum potential set the size of which is established based on the maximum observed frequency for the given household type. Then we allow some activities to have zero parties (i.e. not to be undertaken at all). The transformation of the general participation GEV is trivial and requires only extension of the party sizes.
considered. However, the dimensionality of the model and number of nests become significantly higher compared to the participation model with fixed frequency.

The model generalization to combined frequency-participation choice for the general participation case takes the following form:

$$G(\{Y_{o,}\}) = \alpha \sum_{m=M} \left[ \sum_{n=0}^T \left( \sum_{o \in \Omega_{0,2+}} \frac{n}{N_{o,}} Y_{o,} \right)^\mu \right] + \beta \sum_{j \in J} \left[ \sum_{l=0}^T \left( \sum_{o \in \Omega_{0,1}} \frac{l}{N_{o,}} Y_{o,} \right)^\mu \right].$$  \hspace{1cm} (18)

The same frequency-participation generalization but adjusted to the particular case of the joint participation model results in the following form:

$$G(\{Y_{o,}\}) = \alpha \sum_{m=M} \left[ \sum_{n=0}^T \left( \sum_{o \in \Omega_{0,1}} \frac{n}{N_{o,}} Y_{o,} \right)^\mu \right] + \beta \sum_{j \in J} \left[ \sum_{l=0}^T \left( \sum_{o \in \Omega_{0,2+}} \frac{l}{N_{o,}} Y_{o,} \right)^\mu \right].$$  \hspace{1cm} (19)

The same frequency-participation generalization but adjusted for the particular case of the individual task allocation results in the following form:

$$G(\{Y_{o,}\}) = \sum_{m=M} \left[ \sum_{n=0}^T \left( \frac{n}{J} \sum_{o \in \Omega_{0,1}} Y_{o,} \right)^\mu \right].$$  \hspace{1cm} (20)

However, the feasible matrix sets in all three cases should be reconstructed in order to include empty columns (activities that have not been assigned a participant). It relates to the extension of the general matrix set $\Omega$, matrix set with joint participations $\Omega_{0,2+}$, and matrix set with individual allocations $\Omega_{0,1}$. If activities $j \in J$ are not distinguished by their attributes but only by order (i.e. there is no difference between the 1st and 2nd activity), the set of matrices can be simplified in such a way that it would exclude all matrices where the 1st activity is empty while the 2nd activity is not empty, etc.

Though combining frequency and participation choices in one structure leads to certain technical complications in view of the large and more complicated set of matrices, it eliminates the need in special formulation, estimation, and application of the separate choice-frequency model prior to the participation model with many of the variables being duplicated. The simultaneous structure also guarantees a full integrity between the frequency and participation choices, i.e. activities are undertaken only if there are available and benefiting participants for each of them.

There is also additional aspect of integrity in the formula (18) above if joint and allocated activities are not separated in the model structure as was shown in the Figure 1 above but considered as a simultaneous generation-participation choice. It can be done theoretically, but the following difficulties should be considered:
Joint non-mandatory activities include maintenance and discretionary activity types. It is logical to model joint maintenance activities and individual (allocated) maintenance activities in one simultaneous structure since they may have strong substitution between them. However, this logic cannot be extended to individual discretionary activities. Individual discretionary activities do not assume an intra-household allocation mechanism. Thus they either have to be excluded or considered on a different basis using parallel choice technique. This constitutes an interesting avenue for future research.

There are other choices – ride-sharing and escorting – between the joint and allocated maintenance activities in the general choice hierarchy shown in the Figure 1 above. Their integration with the participation choices is more problematic.

The dimensionality, complexity of the utility expressions, and nesting structure for the simultaneous model are significantly higher compared to the each of the particular models in the sequence. This also results in significant running times when the model is applied for synthetic population of real-world size.

It should be noted, however, that most of these difficulties are technical and will be resolved in the near future. It is important to mention also that the resulting complicated structures with thousand of alternative matrices are treated programmatically at both estimation and application stages. The matrices are listed explicitly and then infeasible alternatives are made unavailable following the predetermined algorithm. The utility expressions are combined from a limited set of components. This model structure has an “interpolation” property in a sense that not all possible matrices have to be observed in order to estimate a full choice model that would handle all possible matrices. Utilities for unobserved matrices are restored as combinations of known components estimated for the observed matrices.

5.3. Partition Choice Structure

The partition choice structure is useful when the decision-making is associated with combining elemental units from a given set $i \in I$ (activity episodes, tours, or half-tours) into mutually exclusive subsets $I_j \in I$. A unique partition of the set $I$ into subsets $I_j$ is denoted $\rho$. Each partition is associated with a set $j \in J_\rho$ of subsets of the elemental units $\{i \in I_j\}_{j \in J_\rho}$. The partition problem constitutes choice of the partition $\rho$ out of the set of possible partitions $\rho \in P$.

The partition choice structure serves as a core model for ride-sharing where individual half-tours for mandatory activities correspond to elemental units $i$ while rides $j$ correspond to subsets $I_j$. The entire daily structure of the household rides constitutes an alternative $\rho$.

The partition choice structure also serves as a core model for escorting where children’s half-tours correspond to elemental units $i$ while escorts $j$ correspond to subsets $I_j$. 
However, in the escort model, partition choice is also combined with task allocation choice described in the previous section since escorts have to be assigned to the chauffeurs \( m \in M_a \).

Partition of a set of objects into sub-sets subject to constraints in a deterministic framework is a well-known cluster-analysis problem. This problem is solved by optimizing a utility function associated with each partition \( V_\rho \) under constraints that express feasibility rules for forming subsets \( \{ i \mid j \in J_\rho \} \). The utility function is frequently assumed as a simple sum over utilities associated with each subset \( V_j \) while the subset utility is frequently based on the pair-wise measure of suitability of two objects to be in the same subset \( V_{ij} \). This simple utility-formation rule can be written in the following way:

\[
V_\rho = \sum_{j \in J_\rho} V_j = \sum_{j \in \mathcal{J}, i \in \mathcal{I}_j} V_{ij}.
\]  

(21)

In the cluster-analysis problem, possible partitions \( \rho \in \mathcal{P} \) are normally not listed explicitly but rather defined by the constraints imposed on the subsets’ size and structure through Boolean variables \( \delta_{ij} = (0,1) \) that define the distribution of the objects by subsets in the following way:

\[
\sum_{j \in J} \delta_{ij} = 1, \quad \forall i \in I,
\]

(22)

\[
I_j = \{ i \mid \delta_{ij} = 1 \}.
\]

(23)

The subsets \( j \in J \) are not related explicitly to partitions \( \rho \in \mathcal{P} \) since some of them are allowed to be empty as well as some other ones can consist of a single object. Thus, formally all partitions \( \rho \in \mathcal{P} \) use the same (maximum) set of subsets \( J \) and the partition-specific set of non-empty subsets \( J_\rho \) is singled out by the variables that correspond to the partition configuration \( \{ \delta_{ij}(\rho) \} \):

\[
J_\rho = \left\{ j \in J \left| \sum_{i \in I} \delta_{ij}(\rho) > 0 \right. \right\}.
\]

(24)

Thus, the cluster-analysis problem can be reduced to an optimization problem with the objective function equal to partition utility. Each possible partition corresponds to a feasible solution from the region specified by the constraints. The optimization problem can have either a unique solution or several solutions that share the same maximum utility. The cluster-analysis problem can handle situations with hundreds or even thousands of elemental objects that result in practically infinite number of possible
partitions. The advantage of the optimization problem formulation is that possible partitions do not have to be listed explicitly.

The partition choice structure has certain similarities to the cluster-analysis problem. It considers the same set of partitions and often the utility function is formed in a similar way, i.e. combining pair-wise utilities by objects. However, there are two important differences between the partition choice and cluster-analysis structures:

- Partition choice explicitly considers all possible partitions \( \rho \in \mathbb{P} \) and evaluates probability \( P(\rho) \) of each partition to be chosen. Cluster analysis considers all possible partitions implicitly through optimization under constraints and only the best partition is explicitly reported and evaluated.

- Partition choice requires explicit enumeration of partitions \( \rho \in \mathbb{P} \) while cluster analysis does not require that. As a result, partition choice can be realistically applied only for a limited number of objects (under 10 where the resulted number of partitions is under 1,000). For the same reason, partition choice normally assumes more restrictive rules in order to reduce the number of possible partitions as much as possible and eliminate alternatives with low probability form the beginning. For example, in the application for ride-sharing, it may be beneficial in practical terms to eliminate all pairs of riders with more than 10 miles of distance deviations from the shortest path since this deviation is not observed. Cluster analysis would eliminate these alternatives through utility optimization process. The partition choice could do the same theoretically, however in practical terms, it is better to be done by restricting the choice set.

The core partition choice model can be generated by the following GEV function:

\[
G(\{Y_\rho\}) = \sum_{i_1, i_2 \in I} \left( \sum_{\rho \in \mathbb{P}_{i_1i_2}} \alpha_{\rho_{i_1i_2}} Y_\rho \right)^\mu + (Y_0)^\mu, \tag{25}
\]

where:
- \( i_1, i_2 \in I \) = all possible pairs of objects that serve as nests,
- \( \rho \in \mathbb{P}_{i_1i_2} \) = partitions in which objects \( i_1 \) and \( i_2 \) belongs to the same subset,
- \( \alpha_{\rho_{i_1i_2}} \) = inclusion coefficients of the partition into nests,
- \( Y_0 \) = special partition with all subsets having one object (no pairs),
- \( 0 < \mu \leq 1 \) = nesting coefficient assumed to be in the unit interval.

This structure constitutes a cross-nested logit model where nests are associated with pairs of objects and partitions are considered as elemental alternatives. The inclusion coefficients are calculated by the following formula:
\[ \alpha_{\rho_1, \rho_2} = \frac{1}{|I_\rho^2|}, \]  

where \( I_\rho^2 \) is a set of object pairs included in one of the partition subsets. The partition with no pairs (each subset includes only one object) is singled out in a special nest. The rational behind this choice structure is that two alternative partitions are treated as similar if they have many mutual pairs of objects (and also unpaired subsets of objects are similar). This logic can be generalized to consider triples, quadruples, and further orders of subsets as upper-level nests.

For an example of ride-sharing shown in the Figure 8 above, the general partition choice formulation can be either applied “as is” or adapted to account for specific features of outbound (morning) and inbound (evening) commuting legs in the following way:

\[
G\left( Y_{\rho_1, \rho_2} \right) = \theta \times \left[ \sum_{i, j \in I} \left( \sum_{\rho_1, \rho_2 \in P_{012}} \alpha_{\rho_1, \rho_2} Y_{\rho_1, \rho_2} \right)^\mu + \left( Y_{0, \rho_2} \right)^\mu \right]^\gamma 
+ (1 - \theta) \times \left[ \sum_{i, j \in I} \left( \sum_{\rho_1, \rho_2 \in P_{012}} \alpha_{\rho_1, \rho_2} Y_{\rho_1, \rho_2} \right)^\mu + \left( Y_{\rho_1, 0} \right)^\mu \right]^\gamma, \]  

where:
- \( \rho_1 \in P \) = outbound ride-sharing structures,
- \( \rho_2 \in P \) = inbound ride-sharing structures,
- \( 0 \leq \theta \leq 1 \) = relative importance of outbound ride-sharing compared to inbound,
- \( 0 < \gamma \leq 1 \) = additional upper-level nesting coefficient.

It also should be noted that though the nested structure is fully separable by outbound and inbound direction, each elemental alternative \( \rho_1, \rho_2 \) constitutes a unique combination of outbound and inbound ride-sharing structures. Thus, each alternative is partially included into appropriate outbound nests and at the same time into inbound nests. Also, each alternative can have interaction terms in the utility function (for example, additional utility of having a ride in both directions discussed above). For these reason the two-dimensional choice of the entire-day ride-sharing structure cannot be reduced to two independent choices by directions. Relative importance relates to the definition of similarities of alternatives. The larger is \( \theta \) the more important are similarities across alternatives in terms of the outbound ride-sharing compared to similarities in terms of inbound ride-sharing.

Escorting choice model structure shown in the Figure 9 above requires a composition of the partition choice and task allocation structures described above. Consider a simultaneous choice of escort tour formation \( I_j \) and allocation to the chauffeurs \( J_m \).
The choice set should include all feasible combinations $\rho_\omega$ of tour formation $\rho \in P$ and tour-allocation-to-chauffeurs $\omega \in \Omega_\rho$ matrices. We also assume that escort tour formation done by a partition choice model (and decision to give child half-tour and escort service or not is part of a partition process) and allocation to chauffeurs is done at the same level of the choice hierarchy by means of the different branches of the nested structure. The following GEV function generates this choice model:

$$G\left(\{Y_{\rho_\omega}\}\right) = \theta \sum_{m \in M} \left( \sum_{n=0}^{7} \left( \sum_{\omega \in \Omega_\rho \setminus \Omega_0} \frac{n}{Y_{\rho_\omega}} \right)^{\mu} \right)^{\nu}$$

$$+ \left( 1 - \theta \right) \left( \sum_{l \in \ell} \left( \sum_{\rho \in P_{m2}} \alpha_{\rho/l_2} Y_{\rho_\omega} \right)^{\mu} + (Y_0)^{\mu} \right)^{\nu},$$

(28)

$$0 \leq \theta \leq 1 \quad \text{relative importance of workload compared to escort tours.}$$

In this structure two alternative escorting structures are considered similar if they are characterized by similar workloads for the chauffeurs as well as similar escort tour structures in terms of what children half-tours is served and what is not and how the served children’s half-tours are combined in escort tours. Relative importance relates to the definition of similarities of alternatives. The larger is $\theta$ the more important are similarities across alternatives in terms of the chauffeurs workload compared to similarities in terms of escort tour structure.

6. Conclusions

The following conclusions can be made to summarize the current stage of the research:

- **Incorporation of intra-household interactions** constitutes an important aspect for further progress in modeling activity and travel-related decisions. Recognition of this importance has recently produced a growing body of research on various aspects of modeling intra-household interactions and group decision making mechanisms as well as first attempts to incorporate intra-household interactions in regional travel demand models.

- The previously published research works were mostly focused on time allocation aspect and less on generation of activity episodes, trips, and travel tours that are necessary units for compatibility with regional travel demand models. Also, most of the approaches were limited to household heads only and did not consider explicitly the other household members as acting agents in the intra-household decision making.

- The proposed approach distinguishes between three principal levels of intra-household interactions: 1) Coordinated principal daily pattern types, 2) Episodic joint activity and travel, 3) Intra-household allocation of maintenance activities. The proposed structure gives a good coverage for most frequent cases of intra-household interactions observed in the household travel surveys. This structure serves as a constructive framework for derivation of operational choice models.
that can be estimated based on available household travel surveys and applied in a framework of regional travel demand models.

- **Coordinated principal daily pattern types** relate to the entire-day level of interactions. We consider three principal daily pattern types: (1) mandatory (work, university or school activities, which might include additional out-of-home non-mandatory activities); (2) non-mandatory travel (only non-mandatory activities at least one of which is out of home); and (3) staying at home or absence from town for the entire day. Statistical evidence shows strong coordination between household members at this principal level, resulting in such decisions as staying home for child care; coordinated work commutes; and household members taking time off together for major shopping trips, family events and vacations.

- **Episodic joint activity and travel** occur even if household members have chosen different pattern types (for example, one mandatory and the other non-mandatory) they may participate in shared activities and/or joint travel arrangements. We propose a classification of typical joint activity and travel types that support the development of operational choice models. In particular, we distinguish fully joint travel tours for shared activities from partially joint tours, in which household members share transportation without participation in the same activity.

- **Intra-household allocation of maintenance activities** represents another important aspect of intra-household interactions. Many of the routine household maintenance activities (shopping, banking, visiting post office, etc) are implemented and scheduled individually; however, generation of such an activity and its allocation to a particular household member is a function of a household decision-making process. Thus, these activities require an intra-household interaction mechanism to be properly understood and modeled. Maintenance task allocation mechanism may not be observed completely within a one-day framework since most of the maintenance tasks have cycles longer than one day.

- **Episodic joint activity and travel** can be further classified by the following three principal categories:
  - **Joint travel generated by the shared activity.** This category is almost exclusively bound to non-mandatory activities (shopping, eating out, other maintenance, and discretionary activities) as well as almost exclusively implies a fully joint tour structure.
  - **Joint travel to synchronized mandatory activities.** This category has a significant share of drop-offs and pick-ups of school children made by workers on the way to and from work. Additionally, a significant percentage of school children travel together to and from school generating fully-joint tours and joint half-tours. Also carpooling of workers for commuting to work is observed, though this type has a comparatively low percentage.
o **Escorting** that is a reported “altruistic” purpose of driving some other household member without participation in the activity. Statistical analysis has shown that majority of escorting is associated with serving children who cannot drive alone and, in the case of preschool children, cannot even ride transit alone.

- It is shown that the choice structures can be reduced to a combination of a limited number of typical models that belong to the Generalized Extreme Value (GEV) class of logit-based models. These models together create an analytical framework for integrative modeling of the daily activity and travel of multiple household members, taking into account their interactions. The following three typical choice structures can be mentioned:

  o **Parallel choice model.** This structure is useful for models like daily activity pattern choice of several household members and other choice situation where joint activities are considered along with individual activities and consideration of joint activities is important only across particular choice dimensions while we can assume independence of individual decisions across other dimensions. This structure ensures that individual choices are consistent with and contribute to the utility of higher level joint choices.

  o **Participation choice model.** This structure is useful for modeling participation in joint activities as well as individual allocation of tasks to the household members. This structure ensures that activities are generated at the household level and then participation choices are made in a consistent way across all activities and persons through intra-household, joint decision making.

  o **Partition choice model.** This structure is useful for modeling ride-sharing and escorting since it is designed to consider alternative pools of activities from a predetermined set. The model is designed to capture intra-household relative preferences in pooling different activities together as well as capturing compromises between convenience for passengers who obtain a ride and chauffeurs who experience additional disutility associated with “workload”.

- The limited size of the paper and the authors’ intention to discuss various types of intra-household interactions and the corresponding modeling structures in a comprehensive way made it impossible to include the results of statistical analysis and model estimation. These results will be presented at the conference.
7. References


