Anticipatory Pricing to Manage “Flow Breakdown”

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• Flow = density × speed
Fundamental diagram of traffic
Fundamental diagram of traffic
“Flow Breakdown”

![Flow Breakdown Graph]

- Flow Breakdown chart showing the relationship between flow (veh/hour) and density (vehicles per lane mile).
- The graph indicates that as density increases, flow also increases, reaching a peak before declining.
- The data points are plotted with red crosses, and a trend line is shown in red.
- The y-axis represents flow in veh/hour, ranging from 0 to 2,500.
- The x-axis represents density in vehicles per lane mile, ranging from 0 to 70.
Causes

• Weaving between lanes
• Excessively slow vehicles
• Aggressive driving
• Sharp brakeing
• Unusual weather
• Unusual visual distraction
Features

• Does not occur everyday (probabilistic)

• Precursor action more likely to result in breakdown at higher densities/flows

• Occurs when highway is operating at less than theoretical capacity
What it is not

• Backup caused by a downstream bottleneck now affecting this link

• Random traffic crashes that close some or all lanes

• Oversaturation of the link
Congestion pricing I

• Flow less than maximum capacity:
  – “normal congestion” (economists)
  – “undersaturated” (engineer)

• Has stable density/speed/flow relationship
Cost = $f(1/\text{speed})$

- AVC
- MC

Flow
Veh/hr
Cost = f(1/speed)

- AVC
- MC
- Demand
- Flow (Veh/hr)
Cost = f(1/speed)
Congestion pricing II

• (In)flow greater than maximum capacity:
  – “hypercongested” (economists)
  – “oversaturated” (engineer)

• “Bottleneck” model
  – Dates to Vickrey in the 1960s
  – Modern version started with Arnott, De Palma and Lindsey, 1990
Congestion pricing II

• Bottleneck of fixed capacity
• If inflow to bottleneck exceeds capacity, then a queue develops
• Drivers suffer a travel time penalty in the queue
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• Drivers suffer a travel time penalty in the queue
• Drivers endogenously select their departure time from “home” (discussed in a minute)
• May arrive at “work” earlier or later than they would like (disutility from this variation)
Congestion pricing II

- Bottleneck of fixed capacity
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- Drivers endogenously select their departure time from “home” (discussed in a minute)
- May arrive at “work” earlier or later than they would like (disutility from this variation)
- Introduction of pricing shortens smooths inflow and makes drivers better off
Our objective

• Adapt the bottleneck model to deal with situations where the equilibrium flow is less than theoretical capacity:
  – “Good days” when drivers encounter no congestion
  – “Bad days” when breakdown occurs (a bottleneck become binding) and drivers encounter congestion in the form of a queue

• Make the probability of a “bad day” endogenous
How are you going to price?

• Option 1 – real time dynamic ex-post pricing to help highway recover on “bad days”
  – Need alternative routes
  – And/or people delay or not make trips or change mode

Dong and Mahmassani (2013)
How are you going to price?

• Option 1 – real time dynamic ex-post pricing to help highway recover on “bad days”
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  – And/or people delay or not make trips or change mode

• Option 1A – upstream sensors and traffic prediction models guess if and where breakdown is likely and price accordingly – Dong and Mahmassani (2013)
How are you going to price?

• Option 2 – Anticipatory pricing:
  – Same price on both good and bad days
  – Price set in advance so drivers know it in making departure time decisions
  – Drivers know in advance how traffic performs on both good and bad days
  – Drivers know the – endogenous – probability of a bad day
  – Hence choose their departure time from home
THE MODEL
Simplifications

• Morning peak
• Fixed “totally inelastic” number of commuters (Q) in single-occupancy cars
• Homogenous drivers (same utility function and tastes)
• Same desired arrival time at work (t*)
Simplifications

• Single link between “home” and “work”
• “Home” is located immediately before a possible bottleneck
• “Work” is located immediately after the bottleneck
• So, free-flow travel time and vehicle operating costs are normalized to zero
Highway technology

• (Out)flow capacity of the bottleneck in non-breakdown state ($V_K$) is not binding on inflow $V_a(t)$ for any value of $t$ on a good day

• If breakdown occurs, capacity falls to $V'_K$, which is binding on $V_a(t)$ for at least some values of $t$

• Then a vertical queue develops

• Highway remains in breakdown state until queue totally dissipates, then it resets
Probability of breakdown

Probability of Breakdown

1

0

Inflow $V_a$

$V_{K'}$ $V_K$
Probability of breakdown

Probability of Breakdown vs Inflow $V_a$

$0 \leq P_{\text{breakdown}} \leq 1$

$V_{K'}$ and $V_K$ are critical points for the probability of breakdown.
Probability of breakdown

Probability of Breakdown

Inflow $V_a$

$0 < V_K < V_K'$
When does breakdown occur?

- We will show that inflow $V_a(t)$ is highest earlier in the peak period
- Breakdown probability based on this maximum inflow
- If breakdown occurs at all, it happens immediately at the start of the peak
- Random drawing each morning based on endogenous probability
- Because on a bad day the queue does not dissipate until the end of peak, cannot have highway “recover” and then face possibility of relapse into breakdown
- So breakdown either at beginning of peak or not at all
Driver decision making

- All desire to get to work at $t^*$
- Chose departure time from home in continuous time $t$
- Departure time can be earlier than, same as, or later than $t^*$
- Objective is to minimize disutility (generalized cost) of their trip
- Equilibrium conditions:
  - No driver can shift departure time to improve their welfare
  - (implies that all drivers face same generalized cost)
  - Everyone who leaves home gets to work!
Generalized cost

• Some things normalized to zero
  – Free-flow travel time
  – Vehicle operating costs

1. Travel time delay (time in queue) valued at $\alpha$
  – Note that on “good day” travel delay is zero

2. Schedule delay - work arrival time relative to $t^*$
  – if arrive at work early valued at $\beta$
  – if arrive at work late valued at $\gamma$
  – usual assumption that $\beta < \alpha < \gamma$

3. Time-varying toll $\tau(t)$
NO-TOLL BASE CASE
Three groups of commuters

- **Early**: arrive at work early or exactly “on time” on both good and bad days
Early commuters

c_g(t) = p(V_a^{early}) \{\alpha T_{DB}(t) + \beta [t^* - (t + T_{DB}(t))]\}
+ [1 - p(V_a^{early})] \beta (t^* - t)
Early commuters

\[ c_g(t) = p(V_a^{\text{early}}) \left\{ \alpha T_{DB}(t) + \beta \left[ t^* - (t + T_{DB}(t)) \right] \right\} 
+ [1 - p(V_a^{\text{early}})] \beta (t^* - t) \]

Solve by:
\[ \frac{\partial c_g(t)}{\partial t} = 0 \]

\[ \frac{\partial T_{DB}(t)}{\partial t} = \frac{V_a^{\text{early}}}{V_{K'}} - 1 \]
Early commuters

\[ V_{a}^{early} = \left[ \frac{\beta}{p(V_{a}^{early})(\alpha - \beta)} + 1 \right] V_{K}, \]
Three groups of commuters

• **Early**: arrive at work early or exactly “on time” on both good and bad days

• **Middle**: arrive at work early or exactly on time on good days and late on bad days
Middle commuters

\[ c_g(t) = p(V_{a}^{\text{early}}) \{\alpha T_{DB}(t) + \gamma [t + T_{DB}(t) - t^*]\} \]
\[ + (1 - p(V_{a}^{\text{early}})) \beta (t^* - t) \]

Solve in similar fashion
Middle commuters

\[ V_{a\text{middle}} = \left[ \frac{(1 - p(V_{a\text{early}})) \beta - p(V_{a\text{early}}) \gamma}{p(V_{a\text{early}})(\alpha + \gamma)} + 1 \right] V_K, \]

- \( V_{a\text{middle}} < V_{a\text{early}} \)
Three groups of commuters

- **Early**: arrive at work early or exactly “on time” on both good and bad days

- **Middle**: arrive at work early or exactly on time on good days and late on bad days

- **Late**: arrive at work late on both good and bad days
Late commuters

c_g(t) = \begin{align*}
p(V_{a^{\text{early}}}) \{& \alpha T_{DB}(t) + \gamma \left[ t + T_{DB}(t) - t^* \right] \} \\
+ \left[ 1 - p(V_{a^{\text{early}}}) \right] \gamma \left( t - t^* \right)
\end{align*}

Solve in similar fashion
Late commuters

\[ V_{a_{late}} = \left[ \frac{-\gamma}{p(V_{a_{early}})(\alpha + \gamma)} + 1 \right] V_{K'} \]

- \( V_{a_{late}} < V_{a_{middle}} < V_{a_{early}} \)
- \( V_{a_{late}} < V_{K'} \)
The model

- **Predetermined parameters**: $Q$, $\alpha$, $\beta$, $\gamma$, $V_K$, $V_K'$, distribution of $p(V_a^{\text{early}})$
- **Just determined**: $V_a^{\text{early}}$, $V_a^{\text{middle}}$, $V_a^{\text{late}}$
- **Still to be determined**:
  - $t_q = \text{departure time of earliest commuter}$
  - $t_{em} = \text{break point between early and middle group}$
  - $(t_{ml} = t^* = \text{break point between middle and late group})$
  - $t_{q'} = \text{departure time of the last commuter}$
  - $Q_{\text{early}}$, $Q_{\text{middle}}$, $Q_{\text{late}}$
Time line (not to scale)

Early  Middle  Late

$t_q$  $t_{em}$  $t^* = t_{ml}$  $t_{q'}$
Can define a system of linear equations to solve for these

\[ t_q = t^* - \frac{Q}{1 + \frac{V_{a, middle}}{V_{a, late} - V_{K'}} \left( \frac{V_{K'}}{V_{a, early}} - 1 \right)} V_{K'} \]

\[ t_{em} = \frac{V_{K'}}{V_{a, early}} t^* + \left( 1 - \frac{V_{K'}}{V_{a, early}} \right) t_q \]
Can define a system of linear equations to solve for these

\[ T_{DB}(t^*) = \frac{V_{a, middle}}{V_{K'}} t^* - \frac{V_{a, middle}}{V_{K'}} t_{em} \]

\[ t_{q'} = t^* - \frac{V_{K'}}{V_{a, late} - V_{K'}} T_{DB}(t^*) \]

\( Q_{early}, Q_{middle}, Q_{late} \) follow from these
OPTIMAL FINE TOLLS
The standard bottleneck model

• When inflow > capacity even on a good day
• Set price schedule so there is a constant inflow equivalent to the bottleneck capacity (no queuing)
• For each driver the combined:
  – travel delay (eliminated by pricing)
  – schedule delay early or late
  – toll paid
is the same (i.e, the less the schedule delay, the higher the toll)
In our model

- Set price schedule to regulate inflow so it a constant rate equivalent to the maximum expected bottleneck capacity

\[ V_a^1 = \max\{[1 - p(V_a)]V_a + p(V_a)V_{K'}\} \]
New time line (not to scale)

Early    Middle    Late

$t_r$    $t_{em}^{-1}$    $t^*=t_{ml}^{-1}$    $t_{r'}$
Solving the model

• Both the very first (at $t_r$) and very last driver (at $t_{r'}$) pay zero toll, but suffer:
  – Schedule delay early and zero traffic delay (on both good and bad days) for first driver
  – Schedule delay late and a queue (on bad days) for last driver
  – These must be the same in equilibrium, denote as $\delta^1$
Solving the model

• For first driver:

\[
\delta^1 = c_g(t_r) = \beta(t^* - t_r)
\]

\[
= \beta Q \left\{ \gamma + p(V_a^1) \left[ (\alpha + \gamma) \left( \frac{V_a^1}{V_{K'}} - 1 \right) \right] \right\}
\]

• Set toll schedule to increases the travel delay and schedule delay for all drivers to \( \delta^1 \)
Optimal toll schedule $\tau(t)$

Early: $t_r \leq t \leq t_{em}^1$
\[ \delta^1 - \beta(t^*-t) - p(V_a^1)\{(\alpha-\beta)[(V_a^1 / V_{K'}) - 1](t-t_r)\} \]

Middle: $t_{em}^1 < t \leq t^*$
\[ \delta^1 - [1- p(V_a^1)] \beta(t^*-t) - p(V_a^1) \gamma(t-t^*) \]
\[ - p(V_a^1)(\alpha+\gamma)[(V_a^1 / V_{K'}) - 1](t-t_r) \]

Late: $t^* < t \leq t_r$,
\[ \delta^1 - \gamma(t-t^*) + p(V_a^1)(\alpha+\gamma)[(V_a^1 / V_{K'}) - 1](t-t_r) \]
Toll schedule (not to scale)

\[ t_r \quad t_{em}^1 \quad t^* = t_{ml}^1 \quad t_r' \]

Early    Middle    Late
EXTENSIONS
Possible extensions

• Rather than at the start of the peak, breakdown may occur randomly within the peak
• Number of commuters (Q) is elastic
• Highway congestible (travel time increases with flow) even in a non-breakdown state
• Second best coarse toll
FINAL THOUGHTS
Summary and final thoughts

• Model with endogenous breakdown probability

• Get day-to-day travel time variability without stochastic demand

• Model describes reality where you are generally early or on time but occasionally late

• Applicable if departure time precommitted
Thank you

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• Read the draft paper: http://faculty.wcas.northwestern.edu/~ipsavage/440-manuscript.pdf
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