Coordination and Optimization of Urban Mobility in Smart Cities

Prof. Nikolas Geroliminis

Urban Transport Systems Laboratory
LUTS team

Nikolas Geroliminis

Jack Haddad (PostDoc) (CE – Control)

Konstantinos Aboudolas (Post Doc) (EE)

PhD Students

Burak Boyaci (OR)

Mohsen Ramezani (EE)

Yuxuan Ji (CS)

Nan Zheng (CE)

Mehmet Yildirimoglu (CE)

LUTS - URBAN TRANSPORT SYSTEMS LABORATORY
\[
\min_{u_1(t), u_2(t), u_{or,1}(k), u_{or,2}(k)} \int_{t_0}^{t_f} [n_1(t) + n_2(t)] dt + \sum_{k=0}^{K-1} \sum_{i=1}^{L} x_i(k)
\]

**Subject to:**

\[
\frac{dn_{11}(t)}{dt} = \frac{q_{31}(t) + q_{23}(t)}{q_{31}(t) + q_{23}(t) + q_{13}(t)} u_{21}(t) \cdot M_{21}(t) + \hat{q}_{31}(t)
\]

\[
\frac{dn_{12}(t)}{dt} = \frac{q_{12}(t) + q_{13}(t)}{q_{31}(t) + q_{23}(t) + q_{13}(t)} u_{21}(t) \cdot M_{21}(t)
\]

\[
\frac{dn_{22}(t)}{dt} = \frac{q_{32}(t) + q_{12}(t)}{q_{31}(t) + q_{23}(t) + q_{13}(t)} u_{21}(t) \cdot M_{21}(t) + \hat{q}_{32}(t) - \min(M_{12}(t), C_{or,1}(t))
\]

\[
\frac{dn_{23}(t)}{dt} = \frac{q_{13}(t)}{q_{31}(t) + q_{23}(t) + q_{13}(t)} u_{21}(t) \cdot M_{21}(t)
\]

\[
C_{or,i}(t) = (n_{or,i}(\text{max}) - n_{or,i}(t)) / n_{or,i}(\text{max})
\]

\[
u_{or,1}(k), u_{or,2}(k) \leq u_{\text{max}}
\]

\[
n_{13}(t) + n_{13}(t) + n_{23}(t) + n_{23}(t)
\]
Big Challenges of Transportation Science

Non-linear interactions
- Components influence the system and v.v.
- Components adapt
- Causes and effects not proportional - Hysteresis
- Congestion spreads
- Transient states

- Limits to predictability
  - Complex Dynamics
  - Chaotic Behavior
  - Sensitivity

- Limits to Control
  - Phase transitions
  - Instabilities
  - Irreducible randomness
  - Local Optima
  - Observability
\[
\frac{d n_{ij}}{dt} = q_{ij} - \sum_{k=1}^{N} q_{i\rightarrow k}^j + \sum_{k=1}^{N} q_{k\rightarrow i}^j
\]
Heterogeneity
- Spatial and Temporal
- Congestion Level
- Topology
- Modes of transport
- Sensing equipment

Sparse multi-sensor data

Develop travel time field and route choice information

Pictures provided by Armando Bazzani (Un. of Bologna)
WHY MACRO?

- **Humans make choices** in terms of routes, destinations and driving behavior (unpredictability – related to control)

- Not a clear distinction between free-flow and congested traffic states. Empirical analysis of spatio-temporal congestion patterns has revealed additional complexity of traffic states (vast literature)

- Need for real-time hierarchical traffic management schemes (decentralized control might not work for heavily congested systems)

Solution (?): Macroscopic Fundamental Diagram (MFD)

“With four parameters I can fit an elephant, and with five I can make him wiggle his trunk”. JOHN VON NEUMANN
Fundamental Diagram (FD) for a link $i$

- Accumulation: $n_i$ (vehs)
- Travel Production: $P_i$ (veh-km/hr) - VKT
- Output-Trip completion rate (vehs/hr)

3 Regimes

I: Undersaturated
II: Efficient
III: Oversaturated

Growing queues from the downstream link block the arrivals
Theory: Generalization to networks

\[ P = \sum P_i = \sum Q_i(n_i) \approx Q\left(\sum n_i\right) \]

AGGREGATE BEHAVIOR

= SCALED UP VERSION OF LINK BEHAVIOR
Literature

Modeling

• MFD-type curves were initially proposed by Godfrey (1969) and later by Herman and Prigogine (1979), Mahmassani et al (1987), Daganzo (2007).

• Dynamic features of MFD with empirical data was firstly observed in Geroliminis and Daganzo (2008).

• Plenty of recent research (IFFSTAR, France; Northwestern; TU Delft; UC Berkeley; EPFL and others)

Control

• Single Region (Daganzo, 2007), (Ekbatani, 2012), Haddad and Geroliminis (2012)
• Multi-region Perimeter Control (Geroliminis, Haddad Ramezani (2012)
• Route choice (Knoop et al, 2012)
MFD Empirical results

- **Fixed sensors**
  - 500 detectors (Occupancy and Counts per 5min)

- **Mobile sensors**
  - 140 taxis with GPS
    - Time and position (stops, hazard lights etc)

- **Geometric data**
  - (detector locations, link lengths, control, etc.)

Network type and MFD

MFD IS NOT A UNIVERSAL LAW

Regularity conditions that possibly ensure an MFD

(i) a slow-varying and distributed demand
(ii) a redundant network with many route choices
(iii) a homogeneous network with similar links

• An MFD with low scatter
  – locally heterogeneous but macroscopically regular networks
    (e.g. cities with multiple modes)
• An MFD with high scatter
  – Networks with uneven and inconsistent distribution of congestion (e.g. freeways)

Conjecture for networks with an MFD: 

\[ D_{t1} \sim D_{t2} \quad \text{iff} \quad \bar{k}_{t1} = \bar{k}_{t2} \]
Properties of a well-defined MFD

\(d_r(t)\): pdf of individual detectors’ density in region \(r\)

\(Q(t)\) and \(O(t)\): Average network flow and density

\(\{Q(t_1) = Q(t_2) \text{ and } O(t_1) = O(t_2)\}\)

\[\iff d_r(t_1) \sim d_r(t_2).\]

Variance much higher than binomial’s

WHY?

Correlation of link density (propagation)

Geroliminis and Sun (2011) – Tr. Res. Part B
Spatial Variability and Network Capacity
An example of a “bad” MFD

- **Strong hysteresis phenomena in freeway MFDs**

**EXPLANATION 1**
- Different distribution of congestion (onset vs. offset)

Freeway network of Minneapolis (USA)

Geroliminis and Sun (2011) – ISTTT 19
An example of a “bad” MFD

Freeway network of Minneapolis (USA)

EXPLANATION

- Different distribution of congestion (onset vs. offset)
- Synchronized Phase Transitions in individual locations

See also Saberi et al. (2012, 2013)
Multi-region cities

- Partitioning + Control, BUT...

\[ \frac{dn_{ij}}{dt} = q_{ij} - \sum_{k=1}^{N} q^j_{i\rightarrow k} + \sum_{k=1}^{N} q^j_{k\rightarrow i} \]

\[ q^j_{i\rightarrow k} = \begin{cases} 
\min \left( x^*_{ik} \cdot a_{i\rightarrow k}, C_{ik} (n_k) \cdot a_{i\rightarrow k}, \frac{V}{n_i} \cdot \frac{n_{ij}}{l_{ij}} \right), & \text{if } \delta_{i\rightarrow k} \neq 0, \\
0, & \text{otherwise.} 
\end{cases} \]

Control will be discussed later
Network partitioning

1. Initialization (Ncut)
2. Merging
3. Boundary Adjustment

\[ \text{Min } NS_k = \frac{\sum_{A \in C} NS_k(A)}{k} \]

where

\[ NS_k(A) = \frac{NS_k(A, A)}{NS_k(A, B)} \]

Intra-Similarity

Inter-Diisimilarity
Congestion Spreading

Open Question: Dynamic Partitioning

Congestion spreading
Shenzhen case study

20000 taxis
9000 links
12 million population

Ongoing – Collaboration with Prof. Luo
Management Options

CONTROL

Without affecting # trips per mode
- Hierarchical Signal Control

By affecting # trips per mode
- Urban Space Allocation
- Pricing
- Parking

Change
Road Share
Change
Mode Share
Control logic

\[ \beta(k) = \beta(k-1) - K_p [n(k) - n(k-1)] - K_I [n(k) - \hat{n}] \]
Simulation results

Aboudolas and Geroliminis (2013) – TRB
Two-region MFDs control problem

\[
J = \max_{u_{12}(t), u_{21}(t)} \int_{t_0}^{t_1} \left[ \frac{n_{11}(t)}{n_{11}(t) + n_{12}(t)} \cdot G_1(n_1(t)) + \frac{n_{22}(t)}{n_{21}(t) + n_{22}(t)} \cdot G_2(n_2(t)) \right] dt
\]

subject to

\[
\frac{dn_{11}(t)}{dt} = q_{11}(t) + \frac{n_{21}(t)}{n_{21}(t) + n_{22}(t)} \cdot u_{21}(t) \cdot G_2(n_2(t)) - \frac{n_{11}(t)}{n_{11}(t) + n_{12}(t)} \cdot G_1(n_1(t))
\]

\[
\frac{dn_{12}(t)}{dt} = q_{12}(t) - \frac{n_{12}(t)}{n_{11}(t) + n_{12}(t)} \cdot u_{12}(t) \cdot G_1(n_1(t))
\]

\[
\frac{dn_{21}(t)}{dt} = q_{21}(t) - \frac{n_{21}(t)}{n_{21}(t) + n_{22}(t)} \cdot u_{21}(t) \cdot G_2(n_2(t))
\]

\[
\frac{dn_{22}(t)}{dt} = q_{22}(t) + \frac{n_{12}(t)}{n_{11}(t) + n_{12}(t)} \cdot u_{12}(t) \cdot G_1(n_1(t)) - \frac{n_{22}(t)}{n_{21}(t) + n_{22}(t)} \cdot G_2(n_2(t))
\]

0 ≤ n_{11}(t) + n_{12}(t) ≤ n_{1, jam}

0 ≤ n_{21}(t) + n_{22}(t) ≤ n_{2, jam}

u_{12, min} ≤ u_{12}(t) ≤ u_{12, max}

u_{21, min} ≤ u_{21}(t) ≤ u_{21, max}

n_{11}(t_0) = n_{11,0}, n_{12}(t_0) = n_{12,0}, n_{21}(t_0) = n_{21,0}, n_{22}(t_0) = n_{22,0}

- MPC can handle:
  - both state and control constraints
  - multiple-input multiple-output nonlinear systems

- Proper for real-time implementation

Description of results (no errors)
Description of results (with errors)

Table 1: The difference between the trip completion corresponding to MPC and greedy control $10^3$[veh] without errors, with small and large errors in the plant MFDs.

<table>
<thead>
<tr>
<th>Example</th>
<th>without errors $(\alpha_1 = \alpha_2 = 0)$</th>
<th>small errors $(\alpha_1 = \alpha_2 = 0.2)$</th>
<th>large errors $(\alpha_1 = \alpha_2 = 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7791.6 (%22.5)</td>
<td>7847.8 (%22.7)</td>
<td>8112.8 (%23.5)</td>
</tr>
<tr>
<td>2</td>
<td>6536.8 (%17.3)</td>
<td>6530.7 (%17.3)</td>
<td>6556.1 (%17.4)</td>
</tr>
<tr>
<td>3</td>
<td>1789.3 (%4.4)</td>
<td>1733.1 (%4.3)</td>
<td>1670.7 (%4.1)</td>
</tr>
</tbody>
</table>
Biased Demand Errors/Smoothing

Fig. 10. Example 1: biased noise in demand.

Fig. 11. Smoothing control by imposing constraints with: (a) $u_{\text{jump}} = 0.1$, and (b) $u_{\text{jump}} = 0.2$. 
Perimeter control case study

• Melbourne  Sydney

Figure 13. Percentage congestion for 3:00 – 3:30 PM and 3:30 – 4:00 PM
Mixed network-Problem Definition

- Problem Statement:
  - The optimal control problem of a mixed traffic network
  - The goal is to minimize the whole network total delay
  - A receding horizon framework (MPC) is proposed to solve the optimal control problem

- Controllers are:
  - 2 Ramp metering
  - 2 Perimeter controllers
Problem Definition

- The urban regions are modeled with the MFD.
- The traffic dynamics of the freeway are modeled according to the asymmetric cell transmission model (ACTM).
- ACTM (Gomez and Horowitz, 2006) is an extension of Cell Transmission Model (Daganzo, 1994).
Asymmetric Cell Transmission Model

- The unmetered On-ramp flow:
  \[ f_{or,l}(k) = \min[n_{or,i}(k) + M_{i3}(t) \cdot T_k, \zeta_l(\bar{x}_l - x_l(k)), s_{or,i}] \]

- The mass conservation equations of the on-ramp:
  \[ n_{or,i}(k + 1) = \min[n_{or,i}(k) + M_{i3}(t) \cdot T_k - u_{or,i}(k) f_{or,l}(k), n_{or,i,max}] \]

- The mainline flow of the cell:
  \[ f_l(k) = \min[(1 - \beta_l(k)), \nu_l(x_l(k) + \gamma \cdot f_{or,l}(k)), w_{l+1}(\bar{x}_{l+1} - x_{l+1}(k) - \gamma \cdot f_{or,l+1}(k)), F_l(k)] \]

- The exit flow of off-ramp:
  \[ f_{off,l}(k) = \frac{\beta_l(k)}{1 - \beta_l(k)} \cdot f_l(k) \]
  \[ F_l(k) = \min[F_l, \frac{1 - \beta_l(k)}{\beta_l(k)} \cdot f_{off,l}] \]

- The mainline mass conservation:
  \[ x_l(k + 1) = x_l(k) + f_{i-1}(k) + u_{or,i}(k) f_{or,l}(k) - f_l(k) - f_{off,l}(k) \]

\( T_k \): time step size [sec]
\( \zeta_l \): on-ramp flow allocation parameter [-]
\( x_l(k) \): accumulation of cell \( l \) at time step \( k \) [veh]
\( \bar{x}_l \): jam accumulation [veh]
\( s_{or,i} \): maximum flow of on-ramp in region \( i \)
\( \beta_l(k) \): split ratio of off-ramp in cell \( l \) [-]
\( \gamma \): on-ramp flow blending coefficient [-]
\( \nu_l \): normalized free flow speed [-]
\( w_l \): normalized congestion wave speed[-]
\( \bar{f}_l \): mainline capacity
\( \bar{f}_{off,l} \): off-ramp capacity
### Urban state variables:

9 accumulations $n_{ij}$:
- number of vehicles at region $i$ going to $j$, e.g.

$$\frac{dn_{11}(t)}{dt} = \text{input from freeway} + \text{input from region 2} + \text{exogenous demand} - \text{trip completion in region 1}$$

### Trip routes assumption:

1. The freeway can be used at most once during the trip (exit and re-enter is not allowed)
2. There is at most one urban region transfer during the trip

<table>
<thead>
<tr>
<th>O \ D</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
</table>
| 1    | $q_{11} : 1 \rightarrow 1$  
$q_{131} : 1 \rightarrow 3 \rightarrow 1$ | $q_{12} : 1 \rightarrow 2$  
$q_{132} : 1 \rightarrow 3 \rightarrow 2$ | $q_{13} : 1 \rightarrow 3$  
$q_{123} : 1 \rightarrow 2 \rightarrow 3$ |
| 2    | $q_{21} : 2 \rightarrow 1$  
$q_{231} : 2 \rightarrow 3 \rightarrow 1$ | $q_{22} : 2 \rightarrow 2$  
$q_{213} : 2 \rightarrow 1 \rightarrow 3$ | $q_{23} : 2 \rightarrow 3$  
$q_{213} : 2 \rightarrow 1 \rightarrow 3$ |
| 3    | $q_{31} : 3 \rightarrow 1$  
$q_{321} : 3 \rightarrow 2 \rightarrow 1$ | $q_{32} : 3 \rightarrow 2$  
$q_{312} : 3 \rightarrow 1 \rightarrow 2$ | $q_{33} : 3 \rightarrow 3$ |

Ramezani, Haddad, Geroliminis - Macroscopic Traffic Control of Mixed Arterial-Freeway Networks
Mixed traffic network modeling

- **Q_{ij}(t) [veh/sec]**:
  the total demand generated in origin \( i \) with destination \( j \) at time \( t \), \( i, j = 1, 2, 3 \)

- **Dynamic trip route choice:**
  Based on the current estimation of each trip route travel time, e.g. for O-D (11):

  \[
  t_{t11} = \frac{\text{trip length } 1 \rightarrow 1}{V_1(t)}
  \]

  \[
  t_{t131} = \frac{\text{trip length } 1 \rightarrow \text{on ramp} 1}{V_1(t)} + t_{\text{or,1}(t)} + t_{\text{cells}\{\text{on ramp} 1, \ldots, \text{off ramp} 1\}(t)}
  \]

  \[
  \theta(t) = \Pr(t_{t11}(t) < t_{t131}(t)), \theta(t) \in [0,1]
  \]

  \[
  q_{ij}(t) = \theta(t). Q_{ij}(t)
  \]

  \[
  q_{ikj}(t) = (1 - \theta(t)). Q_{ij}(t)
  \]

\[
\begin{array}{c|ccc}
O \backslash D & 1 & 2 & 3 \\
\hline
1 & q_{11} : 1 \rightarrow 1 & q_{12} : 1 \rightarrow 2 & q_{13} : 1 \rightarrow 3 \\
   & q_{131} : 1 \rightarrow 3 \rightarrow 1 & q_{132} : 1 \rightarrow 3 \rightarrow 2 & q_{133} : 1 \rightarrow 2 \rightarrow 3 \\
2 & q_{21} : 2 \rightarrow 1 & q_{22} : 2 \rightarrow 2 & q_{23} : 2 \rightarrow 3 \\
   & q_{231} : 2 \rightarrow 3 \rightarrow 1 & q_{232} : 2 \rightarrow 3 \rightarrow 2 & q_{233} : 2 \rightarrow 1 \rightarrow 3 \\
3 & q_{31} : 3 \rightarrow 1 & q_{32} : 3 \rightarrow 2 & q_{33} : 3 \rightarrow 3 \\
   & q_{321} : 3 \rightarrow 2 \rightarrow 1 & q_{332} : 3 \rightarrow 1 \rightarrow 2 & q_{333} : 3 \rightarrow 3 \\
\end{array}
\]
Problem Formulation

\[ J = \min_{u_{12}(t), u_{21}(t)} \int_{t_0}^{t_f} [n_1(t) + n_2(t)] dt + \sum_{k=0}^{K-1} \sum_{l=1}^{L} x_l(k) \]

Subject to:

\[ \frac{dn_{11}(t)}{dt} = \frac{\hat{q}_{311}(t) + q_{23}(t)}{\hat{q}_{311}(t) + q_{13}(t) + q_{21}(t)} u_{21}(t) \cdot M_{21}(t) + q_{11}(t) + \hat{q}_{231}(t) + \hat{q}_{311}(t) - M_{11}(t) \]

\[ \frac{dn_{12}(t)}{dt} = q_{12}(t) + q_{123}(t) + \hat{q}_{312}(t) - u_{12}(t) \cdot M_{12}(t) \]

\[ \frac{dn_{13}(t)}{dt} = \frac{q_{212}(t)}{\hat{q}_{312}(t) + q_{212}(t) + q_{21}(t)} u_{21}(t) \cdot M_{21}(t) + q_{13}(t) + q_{131}(t) + q_{132}(t) - \min(M_{13}(t), C_{or,1}(t)) \]

\[ \frac{dn_{21}(t)}{dt} = q_{21}(t) + q_{213}(t) + \hat{q}_{321}(t) - u_{21}(t) \cdot M_{21}(t) \]

\[ \frac{dn_{22}(t)}{dt} = \frac{\hat{q}_{312}(t) + q_{12}(t)}{\hat{q}_{312}(t) + q_{12}(t) + q_{123}(t)} u_{12}(t) \cdot M_{12}(t) + q_{22}(t) + \hat{q}_{132}(t) + \hat{q}_{32}(t) - M_{22}(t) \]

\[ \frac{dn_{23}(t)}{dt} = \frac{q_{123}(t)}{\hat{q}_{312}(t) + q_{12}(t) + q_{123}(t)} u_{12}(t) \cdot M_{12}(t) + q_{23}(t) + q_{231}(t) - \min(M_{23}(t), C_{or,2}(t)) \]

\[ C_{or,i}(t) = (n_{or,i,\max} - n_{or,i}(k))/T_k \; ; \text{available flow capacity in the on-ramp queue} \]

\[ u_{\min} \leq u_{12}(t), u_{21}(t) \leq u_{\max} \; ; u_{\min} \leq u_{or,1}(k), u_{or,2}(k) \leq u_{\max} \]

\[ 0 \leq n_1(t) \leq n_{1,jam} \; ; n_1(t) = n_{11}(t) + n_{12}(t) + n_{13}(t) \]

\[ 0 \leq n_2(t) \leq n_{2,jam} \; ; n_2(t) = n_{21}(t) + n_{22}(t) + n_{23}(t) \]

\[ M_{ij} = \frac{n_{ij}}{n_i} \times G_i(n_i(t)) \]

and ACTM formulas
Proposed Controllers

1. **ALINEA:**
   \[ f_{or,l}(k + 1) = f_{or,l}(k) + \kappa \times (x_l(k) - x_{ref}) \]

2. **ALINEA + Queue Constraint**
   \[ f_{or,l}(k + 1) = \begin{cases} f_{or,l}(k) + \kappa \times (x_l(k) - x_{ref}) & \text{if } n_{or,l} < n_{or,l,\text{max}} \\ s_{or,l} & \text{otherwise} \end{cases} \]

3. **Urban MPC + ALINEA**

4. **Urban MPC + ALINEA (Q)**

5. **Decentralized MPC (Urban MPC + Freeway MPC)**

6. **Cooperative Decentralized MPC (Urban MPC + Freeway MPC)**

7. **Centralized MPC (Network MPC)**
## Results

### Delay (veh.sec X 10^6)

<table>
<thead>
<tr>
<th>Region</th>
<th>ALINEA</th>
<th>ALINEA Q</th>
<th>Urban MPC + ALINEA</th>
<th>Urban MPC + ALINEA Q</th>
<th>D-MPC</th>
<th>CD-MPC</th>
<th>MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>524</td>
<td>438</td>
<td>504</td>
<td>262</td>
<td>537</td>
<td>274</td>
<td>196</td>
</tr>
<tr>
<td>Region 2</td>
<td>121</td>
<td>97</td>
<td>162</td>
<td>160</td>
<td>279</td>
<td>133</td>
<td>143</td>
</tr>
<tr>
<td>Freeway</td>
<td>129</td>
<td>172</td>
<td>129</td>
<td>189</td>
<td>124</td>
<td>213</td>
<td>218</td>
</tr>
<tr>
<td>On-Ramp 1</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>15</td>
<td>16</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>On-Ramp 2</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Network</td>
<td>803</td>
<td>736</td>
<td>823</td>
<td>642</td>
<td>972</td>
<td>642</td>
<td>581</td>
</tr>
</tbody>
</table>

- one hour morning peak simulation plus half an hour post-peak condition
- $N_p = 20$
- $u_{min} = 0.1$, $u_{max} = 0.9$
Alinea vs. Alinea Q

<table>
<thead>
<tr>
<th>Region 1</th>
<th>Region 2</th>
<th>Freeway</th>
<th>On-Ramp 1</th>
<th>On-Ramp 2</th>
<th>Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>524</td>
<td>121</td>
<td>129</td>
<td>13</td>
<td>16</td>
<td>803</td>
</tr>
<tr>
<td>438</td>
<td>97</td>
<td>172</td>
<td>13</td>
<td>16</td>
<td>736</td>
</tr>
</tbody>
</table>

Ramezani, Haddad, Geroliminis - Macroscopic Traffic Control of Mixed Arterial-Freeway Networks
$$tt_{11} = \frac{\text{trip length } 1 \to 1}{V_1(t)}$$

$$tt_{131} = \frac{\text{trip length } 1 \to \text{onramp 1}}{V_1(t)} + tt_{or,1}(t) + tt_{\text{cells\{onramp 1, ..., off ramp 1\}}}(t)$$

$$\theta(t) = \Pr(tt_{11}(t) < tt_{131}(t)), \theta(t) \in [0,1]$$
Multimodal networks

- In urban networks, buses usually share the same network with the other vehicles.
- Movement Conflicts in multi-modal urban traffic systems
- Bus stops affect the system like variable red signals in a single lane (instead of blocking all lanes).
- Increasing bus frequency decreases the flow of vehicles but can increase the flow of passengers.

### Performance Measures

- **Mobility** (Accessibility)
  - Vehicle Hours Traveled
  - Vehicle Kilometers Traveled
- **Emissions** (Environ. Impacts)
- **Costs** (Users, Providers, etc.)
- **Road Space Used**

### MULTIMODAL CITIES

- Competing modes
- Parking
- Pax vs. veh throughput
Multimodal multi-reservoir system

- **Model**: Dynamics
- **Monitor**: Existence + Observability
- **Control**: Redistribution of urban space between modes
Challenging Research Questions

- Control of more complex city structures with route choice
- Field test Implementations
- Congestion Spreading in 2D urban networks
- Travel time distribution estimation with incomplete information
- Dynamic Partitioning Algorithms for urban networks
- Multimodal city networks
DISCUSSION