Pareto Improvements from Lexus Lanes
The case for pricing a portion of the lanes on congested highways

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Traffic congestion is a problem we know how to solve

Costs of traffic congestion

- **52 hr/commuter/yr in major urban areas** (Schrank et al. 2011)
- **1.4% of annual gasoline consumption** (Schrank et al. 2011; EIA 2012)
- Pollution responsible for **8,600 pre-term births** (Currie and Walker 2011)

Solution

- Tolls
- First proposed by Pigou in 1920
A barrier to congestion pricing is the belief that it hurts many road users

▶ Academics
“First-best congestion pricing . . . introduces severe disparities in direct welfare impact.”
Small, Winston, and Yan, 2005

▶ Policy makers
“[Congestion pricing is] unfair in terms of the economic impact.”
Gov. Parris Glendening (D)

▶ Pundits
“Exalted [toll] lanes leave the average Joe in the dust.”
Marc Fisher, The Washington Post

▶ Public
“Turkeys don’t vote for Christmas and motorists won’t vote for more taxes to drive.”
Voter in Manchester, UK
Intuition on congestion pricing supports this conclusion

1. Each car on the road slows others down
Intuition on congestion pricing supports this conclusion

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2. Apply a tax to internalize externality
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5. Faster travel times for those remaining
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- **Kaldor-Hicks** improvement—winners gain more than the losers lose
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2. Apply a tax to internalize externality
3. This raises cost of driving at rush hour
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- **Kaldor-Hicks** improvement—winners gain more than the losers lose
- Problem: Many road users worse off before revenue is spent
**Key result:** A carefully designed toll on a portion of the lanes can be a Pareto improvement before revenue is spent.
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- Give up some potential Kaldor-Hicks efficiency for a Pareto improvement
- If this allows us to overcome political opposition then we’re trading potential efficiency gains for actual efficiency gains
KEY RESULT: A carefully designed toll on a portion of the lanes can be a Pareto improvement before revenue is spent.

- Give up some potential Kaldor-Hicks efficiency for a Pareto improvement
- If this allows us to overcome political opposition then we’re trading potential efficiency gains for actual efficiency gains
- What allows me to get this new result?
  - Identifying a second externality using insights from traffic engineering literature
  - Extend bottleneck model to include this externality and value pricing
Outline

1. Why throughput falls
2. Extend standard model
3. Estimating distribution of driver preferences
4. Welfare effects of congestion pricing
There are two ways congestion reduces throughput:

1. Queue behind bottleneck blocks off-ramps. For example, throughput on I-880N near San Francisco regularly falls by 25% due to queue spillovers from I-238 (Munoz and Daganzo 2002).

2. Once queue forms, throughput at bottleneck drops. For example, throughput on I-805N at 47th St. in San Diego regularly falls by 12% once a queue forms (Chung et al. 2007).
There are two ways congestion reduces throughput

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  - e.g. throughput on I-880N near San Francisco regularly falls by 25% due to queue spillovers from I-238 (Munoz and Daganzo 2002)
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The additional externality give us a backward bending PPF
Extend work-horse model of dynamic congestion to capture additional externality

Bottleneck model (Vickrey, 1969; Arnott, de Palma, and Lindsey 1990)

- Road network

  Home — Work

- Can costlessly split this road into two routes: one priced and one free
Extend work-horse model of dynamic congestion to capture additional externality

**Bottleneck model** (Vickrey, 1969; Arnott, de Palma, and Lindsey 1990)

- Road network
  
  ![Road network diagram]

  - Can costlessly split this road into two routes: one priced and one free

- Congestion
  
  - Only source of delay is a bottleneck of finite capacity
  
  - Only $s^*$ vehicles can pass through bottleneck per minute
  
  - Once queue forms throughput falls to $s < s^*$
Extend work-horse model of dynamic congestion to capture additional externality

**Bottleneck model** (Vickrey, 1969; Arnott, de Palma, and Lindsey 1990)

- Preferences
  - trip price = \( \alpha(\text{travel time}) + \beta(\text{time early}) + \gamma(\text{time late}) + \text{toll} \)
  - Drivers choose
    - Whether to travel
    - Time of departure
    - Route

- Demand \( N(\text{trip price}) \)
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**Bottleneck model** (Vickrey, 1969; Arnott, de Palma, and Lindsey 1990)

- Preferences
  - Trip price = $\alpha$(travel time) + $\beta$(time early) + $\gamma$(time late) + toll
  - Drivers choose
    - Whether to travel
    - Time of departure
    - Route
- Demand $N($trip price$)$
- Equilibrium concept: Pure strategy Nash
## Extend work-horse model of dynamic congestion to capture additional externality

### Bottleneck model (Vickrey, 1969; Arnott, de Palma, and Lindsey 1990)

- **Preferences**
  - trip price = $\alpha(\text{travel time}) + \beta(\text{time early}) + \gamma(\text{time late}) + \text{toll}$
  - Drivers choose
    - Whether to travel
    - Time of departure
    - Route
- **Demand** $N(\text{trip price})$
- **Equilibrium concept**: Pure strategy Nash
- **Atomistic drivers**
- **Continuous time**
Tolls can prevent throughput drop by smoothing departures.
Tolls can prevent throughput drop by smoothing departures

![Graph showing the relationship between departure rate and time of day](image)

- **Departure rate (veh/min)**
  - 48
  - 40
  - 32
  - 8

- **Time of day**
  - 7:00
  - 8:30
  - 9:20

- **r(t)**
- **Maximum throughput**
- **Actual throughput**

The graph illustrates the departure rate over time, with the following observations:

- At 7:00, the departure rate is 48 vehicles per minute.
- At 8:30, the departure rate drops to 8 vehicles per minute.
- At 9:20, the departure rate returns to 48 vehicles per minute.

The graph also highlights the maximum throughput and actual throughput, indicating that tolls can prevent the throughput drop by smoothing the departures.
Tolls can prevent throughput drop by smoothing departures
Tolls can prevent throughput drop by smoothing departures

\[ r(t) \]
\[ r'(t) \]
\[ \text{Maximum throughput} \]
\[ \text{Actual throughput} \]

\[ \Rightarrow \] when agents are homogeneous pricing is a Pareto improvement
Tolls can prevent throughput drop by smoothing departures

\[ r(t) \]

\[ r'(t) \]

\[ r''(t) \]

\[ r(t) \] (veh/min)

Departure rate

Time of day

\[ 7:00 \]

\[ 7:40 \]

\[ 8:30 \]

\[ 9:20 \]

\[ 48 \]

\[ 40 \]

\[ 32 \]

\[ 8 \]

\[ \Rightarrow \] when agents are homogeneous pricing is a Pareto improvement
Allowing agents to be heterogeneous will make it harder to obtain a Pareto improvement

What happens when we price the entire road?

- Internalize externality
- Increase speeds and throughput
- Change currency from time to money
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What happens when we price the entire road?

- Internalize externality
- Increase speeds and throughput
- Change currency from time to money

Drivers can react in three ways

- Travel at same time but pay toll
- Travel at less desirable time, increasing schedule delay
- Stop traveling
By only pricing a portion of the lanes we can still get a Pareto improvement

Intuition for value pricing

<table>
<thead>
<tr>
<th>Both lanes free</th>
<th>Value pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lane 1</td>
<td>Lane 2</td>
</tr>
<tr>
<td>Pricing</td>
<td>Free</td>
</tr>
<tr>
<td>Speed</td>
<td>30</td>
</tr>
<tr>
<td>Volume</td>
<td>50%</td>
</tr>
</tbody>
</table>
Drivers can differ on three dimensions

Dimensions of heterogeneity

- How strongly dislike travel time
  - $\alpha_i$ – value-of-time – willingness to pay in $ to reduce travel time

- How strongly dislike being early or late to work
  - $\beta_i/\alpha_i$ – inflexibility – willingness to pay in travel time to reduce time early

- Value of taking trip
  - Each type has a downward sloping demand curve
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- Value of taking trip
  - Each type has a downward sloping demand curve

**We will hold constant**

- Desired time of arrival
- Ratio of cost of being late to being early

$\gamma_i = \xi \beta_i$ for all types $i$
To make the problem interesting drivers must differ in at least two ways

Must have heterogeneity in

▶ value-of-time

▶ AND one of the following
  ▶ inflexibility,
  ▶ value of taking trip,
  ▶ desired time of arrival, or
  ▶ ratio of cost of being late to being early

otherwise congestion pricing is always a Pareto improvement.
Goal is to estimate parameters so we can evaluate counterfactuals
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What we need to know

- Joint distribution of inflexibility and value-of-time
- Ratio of cost early to cost late
- Elasticity of demand
- Length of rush hour
- Size of throughput drop
Goal is to estimate parameters so we can evaluate counterfactuals

What we need to know

- Joint distribution of inflexibility and value-of-time
- Ratio of cost early to cost late
- Elasticity of demand – Use value from literature: $-0.5$
- Length of rush hour
- Size of throughput drop – Use value from literature: 10%
Goal is to estimate parameters so we can evaluate counterfactuals

What we need to know

- Joint distribution of inflexibility and value-of-time
  - Estimate
    - Marginal distribution of inflexibility
    - Marginal distribution of value-of-time
    - Correlation of inflexibility and value-of-time
  - Combine using copula
- Ratio of cost early to cost late
- Elasticity of demand
- Length of rush hour
- Size of throughput drop
Estimate marginal distribution of inflexibility by matching empirical evolution of travel time across day

- Using theory there is a unique map between
  - distribution of inflexibility,
  - ratio of cost late to cost early,
  - length of rush hour, and
  - desired time of arrival

  and evolution of travel time across day.

- Match using GMM
- Assume inflexibility has a beta distribution
Estimate marginal distribution of inflexibility by matching empirical evolution of travel time across day

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  - distribution of inflexibility,
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and evolution of travel time across day.

- Match using GMM
- Assume inflexibility has a beta distribution
- Data
  - Caltrans PeMS database
  - 5-minute level observations of speed and volume on all CA highways
  - Roughly a detector every mile
  - Construct travel times for median commute
Distribution of flexibility

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflexibility</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.017</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0056</td>
</tr>
<tr>
<td>Ratio of cost late to early</td>
<td>0.58</td>
</tr>
<tr>
<td>Length of rush hour when road free (hrs)</td>
<td>7.33</td>
</tr>
</tbody>
</table>
### Estimating marginal distribution of value-of-time

#### Two approaches

- Use existing estimates
  - Match reported moments of distribution to log-normal
- Use survey data road users on CA SR-91 from Sullivan (1999)
  - Fit household income to a log-normal distribution by MLE
  - Convert to value-of-time by
    - Assume household income comes from driver working 2,000 hours
    - Assume value-of-time is 1/2 wage
Parameter estimates roughly similar to existing estimates

Parameter estimates for distribution of value-of-time

<table>
<thead>
<tr>
<th></th>
<th>MLE</th>
<th>Small et al. (2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median value-of-time</td>
<td>14.63</td>
<td>21.46</td>
</tr>
<tr>
<td>Standard deviation value-of-time</td>
<td>9.026</td>
<td>8.465</td>
</tr>
</tbody>
</table>
“Rich” are less likely to leave early or late to avoid traffic

Probability that a driver left early or late to avoid traffic by income
Can estimate rank correlation between inflexibility and value-of-time

Framework for estimating correlation between inflexibility and value-of-time

- Model predicts inflexible drivers leave closer to peak than flexible drivers
- Rank correlation requires fairly innocuous assumptions
  - Higher income $\Rightarrow$ higher value-of-time
  - Leaving closer to peak $\Rightarrow$ more inflexible
Can estimate rank correlation between inflexibility and value-of-time

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- Model predicts inflexible drivers leave closer to peak than flexible drivers
- Rank correlation requires fairly innocuous assumptions
  - Higher income $\Rightarrow$ higher value-of-time
  - Leaving closer to peak $\Rightarrow$ more inflexible

Data

- Survey questions on time of departure and household income for CA SR-91 road users (Sullivan, 1999)
- Restrict sample to those who living in Riverside or Corona to force commute to be similar
- Only look at early departures to avoid needing to compare early departures to late
Correlation is positive

Correlation between inflexibility and value-of-time

<table>
<thead>
<tr>
<th></th>
<th>Peak at 6:45 am</th>
<th></th>
<th>Peak at 7:00 am</th>
<th></th>
<th>Peak at 7:15 am</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Corr</td>
<td>p-value</td>
<td>Corr</td>
<td>p-value</td>
<td>Corr</td>
<td>p-value</td>
</tr>
<tr>
<td>Kendall’s $\tau-b$</td>
<td>.23</td>
<td>.022</td>
<td>.19</td>
<td>.038</td>
<td>.20</td>
<td>.027</td>
</tr>
<tr>
<td>Spearman’s $\rho$</td>
<td>.30</td>
<td>.020</td>
<td>.24</td>
<td>.037</td>
<td>.26</td>
<td>.026</td>
</tr>
<tr>
<td>N</td>
<td>60</td>
<td></td>
<td>74</td>
<td></td>
<td>75</td>
<td></td>
</tr>
</tbody>
</table>
## Parameter values in base case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflexibility</strong></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>Beta(788.7, 3797)</td>
</tr>
<tr>
<td>Mean</td>
<td>.017</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>.0056</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.58</td>
</tr>
<tr>
<td><strong>Value-of-time</strong></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>In $\mathcal{N}(2.683, .5078)$</td>
</tr>
<tr>
<td>Mean</td>
<td>14.63</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>9.026</td>
</tr>
<tr>
<td><strong>Copula</strong></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>Gaussian(.29)</td>
</tr>
<tr>
<td>Elasticity of demand</td>
<td>-0.5</td>
</tr>
<tr>
<td>Length of rush hour when road free (hrs)</td>
<td>7.33</td>
</tr>
<tr>
<td>Throughput drop (%)</td>
<td>10</td>
</tr>
</tbody>
</table>
### Aggregate welfare effects

<table>
<thead>
<tr>
<th></th>
<th>All Free</th>
<th>Price All</th>
<th>Price 1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction prior road users better off</td>
<td>.99</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Largest price increase ($)</td>
<td>0.023</td>
<td>-0.056</td>
<td></td>
</tr>
<tr>
<td>Toll revenue ($ per capita)</td>
<td>2.7</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>Annual welfare gains ($ per capita)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social</td>
<td>2000</td>
<td>1300</td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>890</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>Private for prior road users</td>
<td>900</td>
<td>440</td>
<td></td>
</tr>
<tr>
<td>Private for prior road users (median)</td>
<td>470</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>Variable travel time (min)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>14</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>Peak</td>
<td>28</td>
<td>0</td>
<td>27</td>
</tr>
<tr>
<td>Toll ($)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0</td>
<td>2.7</td>
<td>4.1</td>
</tr>
<tr>
<td>Peak</td>
<td>0</td>
<td>7.3</td>
<td>10</td>
</tr>
<tr>
<td>Increase in number of drivers (%)</td>
<td>4.7</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>
Pricing all of the road helps the rich but hurts the poor.

![Graph showing the relationship between Inflexibility and Value of Time with varying estimates. The graph illustrates how pricing all of the road affects different economic groups, highlighting the disparity between the rich and the poor.](image-url)
Pricing half the road helps all road users
Value pricing allows the inflexible poor to still travel at peak

<table>
<thead>
<tr>
<th>Inflexibility</th>
<th>Value of Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.160</td>
<td>0</td>
</tr>
<tr>
<td>0.165</td>
<td>1</td>
</tr>
<tr>
<td>0.170</td>
<td>2</td>
</tr>
<tr>
<td>0.175</td>
<td>3</td>
</tr>
<tr>
<td>0.180</td>
<td>4</td>
</tr>
<tr>
<td>0.185</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Price All

(b) Value Pricing
Results are decently robust

- Changing any one parameter doesn’t qualitatively change results.
  - Major exception: > 3\times standard deviation of inflexibility \Rightarrow can’t price > 25\% of road and get Pareto improvement
- Still working on further robustness checks
Conclusion

- Congestion pricing can increase highway throughput
- And so value pricing is likely to be a Pareto improvement before redistributing revenue
- And pricing the entire road is a Pareto improvement as long as a small fraction of the revenue is redistributed
- Welfare gains from pricing entire road are on the order of $2,000 per capita per year, with about half that going to road users
- Pricing a half of the road generates two-thirds of the total welfare gains, but only a third now go to road users
Minneapolis ramp metering experiment

- Turned off ramp meters for 2 months
- During peak periods:
  - Traffic volumes fell 14%
  - Speeds fell 18 mph
  - Travel times increased 22%

Figure: Capacity increase on I-94E due to ramp metering. Reprinted from Mn/DOT, Twin Cities Ramp Meter Evaluation, 2001.
Appendix

No toll equilibrium – ADL (1990)

Figure: No toll equilibrium
No toll equilibrium – ADL (1990)

\[ N(x_0, t) = \left(1 + \frac{\beta}{\alpha - \beta}\right)^s (1 - \frac{\gamma}{\alpha + \gamma})^s N(x_e, t) \]

Figure: No toll equilibrium
No toll equilibrium – ADL (1990)

Figure: No toll equilibrium
Toll equilibrium – ADL (1990)

\[ N(x_0, t) = N(x_e, t) \]

**Figure:** Toll equilibrium
Proposition (Vickrey, 1969)

In the standard bottleneck model with identical agents, optimal road pricing does not change consumer welfare or arrival times, but does raise revenue and reduce travel time.

Figure: No toll equilibrium

Figure: Toll equilibrium
Free vs toll – Throughput drop

Figure: No toll equilibrium

\[ N(x_0, t) = N(x_e, t) \]

Figure: Toll equilibrium

\[ N(x_0, t) = N(x_e, t) \]
Value pricing leads to Pareto improvement

**Proposition**

*Value pricing with identical agents improves welfare for all agents.*

*Furthermore:*

- *the percentage decrease in* $\bar{p} \approx \theta \cdot \lambda$, *and*
- *the percentage decrease in social cost* $\approx \frac{1+\theta}{2} \cdot \lambda$. 
Value pricing – Throughput drop

\[ N(x_0, t) = N(x_e, t) \]

Figure: Free lane

Figure: Priced lane
Definition of equilibrium I

Given a bottleneck congestion game specified by

\[
\left( \{ \alpha_i, D_i (\cdot), t^*_i, N_i (\cdot) \}_{i=1}^G, \{ s, s^*, \bar{T} \}, \{ \lambda_{\text{free}}, \lambda_{\text{toll}} \} \right)
\]

a trip price vector \( \mathbf{p} = (\bar{p}_1, \ldots, \bar{p}_G) \), a set of arrival times \( \{ T_{i,\text{free}}, T_{i,\text{toll}} \}_{i=1}^G \), a set of departure rates \( \{ r_{i,\text{free}}(t), r_{i,\text{toll}}(t) \}_{i=1}^G \), and toll schedules \( \{ \tau_{\text{free}}(t), \tau_{\text{toll}}(t) \} \) constitute a value pricing equilibrium if

1. For every \( i \in \{1, 2, \ldots, G\} \) all agents minimize their trip price; that is,

\[
p_{i,r} (t_a) = \bar{p}_i \quad \text{for} \ t_a \in T_{i,r}, \ r \in \{ \text{free, toll} \}, \ \text{and} \quad (1)
\]

\[
p_{i,r} (t) \geq \bar{p}_i \quad \text{for} \ t \in \bar{T}, \ r \in \{ \text{free, toll} \}. \quad (2)
\]
Definition of equilibrium II

2. For every $i \in \{1, 2, \ldots, G\}$ there is enough time for all $N_i(\bar{p}_i)$ agents to travel, and no more; supply of travel time equals demand for travel time. That is,

$$N_i(p_i) = s \sum_{r \in \{\text{free, toll}\}} |\{ t_a | t_a \in T_{i,r}, \text{ and } Q_r(t) > 0 \text{ or } r_{i,r}(t) > s^* \}|$$

$$+ s^* \sum_{r \in \{\text{free, toll}\}} |\{ t_a | t_a \in T_{i,r}, Q_r(t) = 0 \text{ and } r_{i,r}(t) \leq s^* \}|.$$

(3)

3. Tolls are set on the tolled route to maximize social welfare,

$$\tau_{\text{toll}}(t) = \arg \max_{\tau_{\text{toll}}(t)} \left( \sum_{i=1}^{G} \int_{\bar{p}_i}^{\infty} N_i(p) \, dp + \int_{t \in \bar{T}} \tau_{\text{toll}}(t) \sum_{i=1}^{G} r_{i,\text{toll}}(t) \, dt \right).$$

4. No toll is charged on the free route,

$$\tau_{\text{free}}(t) \equiv 0 \ \forall t \in \bar{T}.$$
Appendix

Change in welfare under congestion pricing when on PPF

\[ T(t_a) \]

\[ \tau(t_a) \]

\[ T^\nu(t_a|p) \]

\[ \tau(t_a|p) \]

\[ \tau(t_a|p') \]

\[ t^* \]

\[ t_0 \]

\[ t_e \]

\[ t_a \]

\[ t'_0 \]

\[ t'_e t_a \]

\[ \tau \]