Traffic congestion in networks, and alleviating it with public transportation and pricing

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References


Outline

• Building blocks
  – City-wide traffic model
  – City-wide transit model

• Putting them together
  – How modes should share street space
  – City scale analysis
City-wide traffic: MFD evidence

- MFD
  - Network flow ($q$) vs. network density ($k$);
    $$q = Q(k)$$

Source: Geroliminis and Daganzo (2008)
City-wide traffic:

Trip completion evidence

- Ratio of trip completion rate ($\mu$) to network flow ($q$)

Source: Geroliminis and Daganzo (2008)
City-wide traffic: Network Exit Function

- Exit flow, $\mu = qL/d$; accumulation, $n = kL$;
- NEF: $\mu = F(n)$

Source: Geroliminis and Daganzo (2008)
City-wide traffic:
What if drivers do not adapt

Source: Daganzo et al (2011a)
City-wide traffic:
Effect of non-adaptive route choice

Source: Gayah and Daganzo (2011)
City-wide traffic: Effect of non-adaptive route choice

Source: Gayah and Daganzo (2011)
City-wide traffic: 
Effect of non-adaptive route choice

Source: Gayah and Daganzo (2011)
City-wide traffic:
Effect of non-adaptive route choice

Source: Gayah and Daganzo (2011)
City-wide traffic: Analytical model (non-adaptive)

Source: Daganzo et al (2011a)
City-wide traffic: Analytical model (adaptive)

Source: Daganzo et al (2011a)
City-wide traffic: Analytical model (adaptive)

Source: Daganzo et al (2011a)
City-wide traffic: Adaptive and non-adaptive cases

Source: Gonzales et al (2009)
City-wide traffic: Spatial considerations

- $l_c$ – space-time consumed by one car trip
- $L_c$ – required network length

$\begin{align*}
L_c &= l_c \mu \\
\text{space needed} &\times \text{time needed} &\times \text{exit rate} \\
\left( \frac{1}{k} \right) \times \left( \frac{d}{v} \right) \times \mu
\end{align*}$

Source: Daganzo et al (2011b)
City-wide traffic: Summary

• Determinants of performance
  – $N_c$ – number of daily car users
  – $\mu$ – exit rate

\[
Z_C(N_C) = z_C N_C
\]
\[
L_C(\mu) = l_C \mu
\]

Source: Daganzo et al (2011b)
City-wide transit:
Overview

Barcelona (2012)

Source: Estrada et al (2011)
City-wide transit: Analytical Model

![Graph showing cost and space used as functions of headway.]

- **Cost:** $Z_T(N_T) = z_1 + z_2 \sqrt{N_T} + z_3 N_T$
- **Space Used:** $L_T(N_T) = (1 - \alpha)L$

**Source:** Daganzo et al (2011b)
City-wide transit:
Spatial distribution of buses

4 Buses Uncontrolled

4 Buses Controlled

Source: Daganzo and Pilachowski (2011)
City-wide transit: Model verification

• Simulation of Barcelona with real network

• Discrepancies between model and simulation
  • User cost (5%)
  • Agency cost (7.5%)

• Most subcomponent discrepancies (10%)

• All discrepancies due to differences between the real and idealized networks

Source: Estrada et al (2011)
Sharing space:
Overview

• Wish to optimally segregate the two modes
Sharing space:
Self-allocation for single mode (the Vickrey problem)

Equilibrium arrival curve and departure curve allows no one to reduce their travel cost by unilaterally choosing another travel time.
Sharing space:
System optimum allocation for single mode and prices to achieve it

\[
\begin{align*}
\text{price} & \quad \text{departure time} \\
0 & \quad \text{early} \quad \text{B} \quad \text{late} \\
A & \quad C
\end{align*}
\]

\[
\begin{align*}
\text{Cum. Trips} \quad \text{(\# trips)} & \quad W(t) \\
N & \quad D(t) \\
\mu & \quad \lambda \\
0 & \quad N_L = Ne \\
N_e = NL
\end{align*}
\]

\[\text{Total earliness} \quad \text{Total lateness} \quad e - \text{earliness penalty} \quad L - \text{lateness penalty}\]
Sharing space: System optimum allocation for two modes

Source: Gonzales and Daganzo (2011)
Sharing space:
System optimal solution

**Schedule Penalty:**

\[
S(N_T) = \left( t_p - \frac{N_T}{\lambda - \alpha \mu} \right)^2 \frac{\lambda e L (\lambda - \mu)}{2 \mu (e + L)}
\]

**Transit Cost and Car Cost:**

\[
Z_T(N_T) = z_1 + z_2 \sqrt{N_T} + z_3 N_T
\]

\[
Z_C(N - N_T) = z_C \times (N - N_T)
\]

**Total Cost:**

\[
Z(N_T) = S(N_T) + Z_T(N_T) + z_C \times (N - N_T)
\]

**System Optimum:**

\[
N_T^* = \arg \min_{N_T} Z(N_T) \quad Z^* = \min_{N_T} Z(N_T)
\]

*Source: Gonzales and Daganzo (2011)*
Sharing Space:
System optimum properties

• Solution depends on properties of the city
  \((z_1, z_2, z_3, z_C) \rightarrow (N, L, l_C \text{ and } \beta)\)

and design variables
  \((\mu \text{ and } \alpha)\)

• \(N_T^*\) can be:
  - 0 \hspace{1cm} \text{All car}
  - \(0, N\) \hspace{1cm} \text{Mixed}
  - \(N\) \hspace{1cm} \text{All transit}

Source: Gonzales and Daganzo (2011)
Sharing space: Pricing to achieve system optimum

Source: Gonzales and Daganzo (2011)
City scale analysis:
Optimum deployment for peaked demand

\[
L \frac{(z_c - z_3)^2}{l_c z_2}
\]

\[
N \frac{(z_c - z_3)^2}{z_2}
\]

Source: Gonzales and Daganzo (2012)
City scale analysis: Optimum deployment for uniform demand

\[
L \left( \frac{z_c - z_3}{l_c z_2} \right)^2
\]

\[
N \left( \frac{z_c - z_3}{z_2} \right)^2
\]

Result for City with Constant Demand

Result for City with Increasingly Uniform Demand

Source: Gonzales and Daganzo (2012)
Where do we go from here?

- Captive users
- Role of underground metro
- Day-long commute
- Pricing
- Experiments