Anticipating Disaster: Planning for Emergency Logistics Needs

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Evacuation events create demands in shelter locations

- Consumables and non-consumables
- Shelter locations frequently aren’t good “pre-positioning” sites
- Focus on delivery capability in first 72 hours
- Lots of uncertainty about “how much and where”
Core Idea

- **Decide on:**
  - Facility locations and sizes
  - Inventory (how much of what is stored where)

under uncertainty in:

- Demands (how much of what, in what locations)
- Possible damage to supplies
- Post-event network condition

- **Minimize expected costs**
  - Facilities
  - Stock Acquisition
  - Transportation
  - Unmet demand penalties
  - Loss/spoilage of unused material

- Stochastic optimization

- First-stage costs
- Second-stage costs
Model Elements

- Define scenarios (with probabilities) to reflect range of uncertainty
  - How many evacuees in which shelters?
  - How much material needs to be supplied by what time?
  - What supplies have survived the event?

- Include transportation limitations
  - Which shelter locations can be supplied from what potential DC locations within first 12 hours; within first 24 hours?
  - What are facility capacities to load/dispatch trucks?
  - What transportation links may be damaged?

- Constraints on solution effectiveness
  - For example: with probability $\alpha$, all demands will be met on time
A Simple Example

One commodity:
- Cost: 5
- Excess cost: 0.5
- Shortage cost: 60

Two possible facility sizes:
- Small: fixed cost = 150, capacity = 100
- Large: fixed cost = 200, capacity = 150

Links have unlimited capacity in all scenarios.

In scenarios 1 and 2, 50% of any material stored at site of event is destroyed; in scenario 3, any material stored at nodes 1 or 2 is destroyed.
Stochastic Mixed Integer Model

Variables

\( y_{il} \) = 1, if there is a facility of size \( l \) at node \( i \)
0, otherwise

\( r^k_i \) = amount of resource of type \( k \) allocated (stored) at node \( i \)

\( x^{ks}_{ij} \) = amount of resource of type \( k \) shipped through link \((i,j)\) in scenario \( s \)
\[
\min \sum_{i \in N} \sum_{l \in L} F_i y_{il} + \sum_{k \in K} \sum_{i \in N} q^k r_i^k + \sum_{s \in S} P_s \left[ \sum_{(i,j) \in A} \sum_{k \in K} c_{ij}^{ks} x_{ij}^{ks} + \sum_{i \in I} \sum_{k \in K} \left( h_i^k z_i^{ks} + p_i^k w_i^{ks} \right) \right]
\]

Subject to:

(i) Open facilities and facility capacity:
\[
\sum_{k \in K} b^k r_i^k \leq \sum_{l \in L} M_i y_{il} \quad \forall i \in N
\]

(ii) Number of facilities per node:
\[
\sum_{l \in L} y_{il} \leq 1 \quad \forall i \in N
\]

(iii) Flow conservation:
\[
\sum_{j \neq l \in N} x_{ji}^{ks} + \rho^k r_i^k - z_i^{ks} = \sum_{j \neq l \in N} x_{ij}^{ks} + v_i^{ks} - w_i^{ks} \quad \forall i \in N, k \in K, s \in S
\]

(iv) Non-negativity constraints:
\[
y_{il} \in (0, 1) \quad r_i^k, x_{ij}^{ks}, z_i^{ks}, w_i^{ks} \geq 0
\]
Solution

- 1 small facility at node 4 with 100 units of material (i.e., filled to capacity)
- Expected total cost = 1349.5
- 90% of time, we have 50 units of extra material, and 10% of time we’re short 100 units
- Expected marginal cost of adding material = $5 + 0.9 \times 0.5 - 0.1 \times 60 = -0.55 \Rightarrow \text{fill to capacity, but not large enough to cover additional fixed cost for larger facility}$
Reliability Constraints

- Ensure that in at least $100\alpha\%$ of outcomes all demand is met on time

$$\gamma_s = 1, \text{ if scenario } s \text{ is included in } \text{“reliability set”; } 0, \text{ otherwise}$$

$$\sum_{s \in S} P_s \gamma_s \geq \alpha$$

$$w_i^{ks} \leq (1 - \gamma_s)v_i^{ks} \quad \forall i \in N, k \in K, s \in S$$
Solution with $\alpha > 0.9$

- 2 small facilities at nodes 3 and 5 with 100 units of material (i.e., filled to capacity to meet demand in scenario 3)
- Multiple facilities $\Rightarrow$ move closer to demands
- Expected total cost = 1432.5 (6% increase)
Demand Distribution

- Basic tradeoff: costs of acquiring and storing material vs. unmet demand penalties
- Required reliability level puts “floor” under amount of material
Demand Dynamics

- At a given shelter, evacuees arrive over time
- Short term forecast of evacuee numbers and “look ahead” policy on materials drives demands to be met
Supply Constraints

- Increasing distance \(\Rightarrow\) delay between shipment and arrival
- Facilities have constraints on rate of loading/dispatch of trucks
Link Capacity Constraints

- Link damage (by scenario) \( \Rightarrow \) reduction of capacity

\[
\sum_{k \in K} u^k x_{ij}^{ks} \leq U_{ij}^s \quad \forall (i, j) \in A, k \in K, s \in S
\]
Unmet demand penalty is based on area between curves (units and time)
**Case study: North Carolina**

- Hurricane threat
- 33 scenarios based on historical storms to match overall hazard estimates (separate study by Legg, et al., 2010)
- HAZUS-MH used to estimate evacuees by census tract for each scenario
- Li, et al. (2010) used scenarios to determine 50 best shelter locations
- These related studies provide input for logistics analysis
Locations

- Shelter capacities between 750 – 3000 persons
- Each shelter used in average of 8 (out of 33) scenarios – max is 17
- Total evacuees in scenarios: 1018 to 56,630; average is 8637
- 16 candidate DC locations
# Facility Data

<table>
<thead>
<tr>
<th></th>
<th>F (cost)</th>
<th>M (capacity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>$20,000</td>
<td>30,000 ft³</td>
</tr>
<tr>
<td>Medium</td>
<td>$48,000</td>
<td>100,000 ft³</td>
</tr>
<tr>
<td>Large</td>
<td>$150,000</td>
<td>400,000 ft³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>C1 (12 hrs)</th>
<th>C2 (24 hrs)</th>
<th>C3 (48 hrs)</th>
<th>C4 (72 hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Medium</td>
<td>0.15</td>
<td>0.4</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Large</td>
<td>0.1</td>
<td>0.25</td>
<td>0.7</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Solution for \( \alpha = 0.9 \)

<table>
<thead>
<tr>
<th>Index</th>
<th>Facility Locations</th>
<th>Facility Capacity</th>
<th>Consumable Supplies (units)</th>
<th>Non-consumable Supplies (units)</th>
<th>Stored Volume (ft(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Nashville</td>
<td>Small</td>
<td>15,000</td>
<td>0</td>
<td>30,000</td>
</tr>
<tr>
<td>11</td>
<td>Dunn</td>
<td>Large</td>
<td>70,715</td>
<td>39,091</td>
<td>375,976</td>
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<tr>
<td>12</td>
<td>Laurinburg</td>
<td>Small</td>
<td>15,000</td>
<td>0</td>
<td>30,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total = 100,715</td>
<td>39,091</td>
<td></td>
</tr>
</tbody>
</table>
## Solution for $\alpha = 0.95$

<table>
<thead>
<tr>
<th>Index</th>
<th>Facility Locations</th>
<th>Facility Capacity</th>
<th>Consumable Supplies (units)</th>
<th>Non-consumable Supplies (units)</th>
<th>Stored Volume (ft³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sanford</td>
<td>Large</td>
<td>95,115</td>
<td>29,876</td>
<td>369,488</td>
</tr>
<tr>
<td>3</td>
<td>Smithfield</td>
<td>Small</td>
<td>15,000</td>
<td>0</td>
<td>30,000</td>
</tr>
<tr>
<td>6</td>
<td>Nashville</td>
<td>Medium</td>
<td>22,356</td>
<td>9,215</td>
<td>100,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total = 132,471</td>
<td>39,091</td>
<td></td>
</tr>
</tbody>
</table>

Total = 132,471 units of consumable supplies, 39,091 units of non-consumable supplies, and a stored volume of 369,488 ft³. 

[Map of facility locations and index](image-url)
### Solution for $\alpha = 1.0$

<table>
<thead>
<tr>
<th>Index</th>
<th>Facility Locations</th>
<th>Facility Capacity</th>
<th>Consumable Supplies (units)</th>
<th>Non-consumable Supplies (units)</th>
<th>Stored Volume (ft$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sanford Large</td>
<td>108,682</td>
<td>30,439</td>
<td>400,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Smithfield Large</td>
<td>102,557</td>
<td>29,653</td>
<td>383,032</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Nashville Small</td>
<td>8,400</td>
<td>2,200</td>
<td>30,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total = 219,639</td>
<td>39,091</td>
<td></td>
</tr>
</tbody>
</table>
First-Stage Costs

- Facilities and material acquisition
- Most “visible” costs
In this case study, load/dispatch capacity is not constraining the system performance.

Maximum Demand Scenario

In this case study, load/dispatch capacity is not constraining the system performance.
Experiments done on a Linux X86-64 dual quad-core workstation

- CPLEX uses 8 cores in parallel during solution

- $\alpha = 1$ solution: $\sim 10$ minutes

- $\alpha = 0.95$ solution: $\sim 33.6$ hours

Current experiments provide proof-of-concept, but a faster (specialized) solution method is needed
L-shaped Method (LSM)

- Takes advantage of
  - Complete recourse
  - Ease of computation of second stage problems in special cases: k*s network flow problems

- LSM solves the approximation of a stochastic program by using the outer linearization of the expected value of the second stage problem
Outer Linearization

- Approximates nonlinear programs by using slopes
- At every iteration an approximation of the first-stage problem is solved using the relaxation of $H(y,r)$

Optimality cuts

Approximate function:

$$\theta \leq H(y,r)$$
Conclusions

- Location-allocation model with dynamic shipment plan for meeting short-term demands resulting from evacuations
- Two-stage stochastic programming formulation
- Case study: stocking supplies for shelters that aid hurricane victims in NC
- Model allows exploration of policy variables, cost tradeoffs, and effects of constraints
- Computational times suggest the need for special algorithm
Questions