Equilibrium and Efficiency of Traffic Flows in Networks (BMW revisited)

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Martin Beckmann in Part I of his 1956 book, *Studies in the Economics of Transportation*[^1], co-authored with C. Bartlett McGuire [1925-2006] and Christopher B. Winsten [1923-2005], established basic principles for user behavior on congested transportation networks, as well as for optimal network vehicle flows, when user choices are respectively uncoordinated or coordinated. In this seminal work, Beckmann provided a rigorous formulation of both non-cooperative, individual decision-making (also known as user-equilibrium) and centralized decision-making (also known as system-optimal) on a congested network, to determine the resulting origin-destination demand, roadway flows and user costs. The formulation and analysis of these two problems is arguably the first use of the theorem of Kuhn and Tucker (1951) in addressing an unsolved problem of economic theory in which totally new results and insights were obtained (as contrasted with proving known results in a new way).

To accomplish this feat, Beckmann:

- defined a representation of a road network with general link cost-flow functions and conservation of flow constraints at nodes;
- proposed a complementarity relationship for shortest route choices from an origin to a destination: if a route flow is positive, then its route cost must be a minimum, and if a route cost is not a minimum, its route flow must be zero[^2];
- defined the general properties of origin-destination demand as a function of endogenously determined user-equilibrium route costs;
- formulated a concave optimization problem whose solution incorporated the above demand and behavioral route choice properties;
- analyzed and interpreted the properties of this formulation and the related system-optimal formulation for efficient road networks.

Aside from the intuitive concept of node conservation of flow (Kirchhoff’s current law), Beckmann’s formulation was completely original. Although Beckmann was not fully aware of the implications of his findings at the time and for some years afterwards, his formulation provided the foundation for new subfield of transportation network economics, operations research and transportation engineering, as well as important areas of professional practice in the fields of transportation network economic policy, planning and engineering.

At the seminar, Professor Beckmann will reflect upon this research accomplishment. Participants will have an opportunity to gain insights into the work through their questions to the speaker.

[^1]: Available online at [http://cowles.econ.yale.edu/archive/reprints/specpub-BMW.pdf](http://cowles.econ.yale.edu/archive/reprints/specpub-BMW.pdf). The Web of Science showed 527 citations of this book in January 2010. Of these, 277 citations (53%) appeared since January 2000, illustrating the significance of this work today. The citing authors represented 39 countries throughout the world.

[^2]: In 1952, a British traffic scientist, John Wardrop, stated a similar criterion, which may be paraphrased as: all used routes have equal and minimal journey times; however, Wardrop did not provide a mathematical formulation.