Multi-facility Maintenance and Rehabilitation Model with Coordinated Intervention

Pablo L. Durango-Cohen
Assistant Professor, Department of Civil and Environmental Engineering,
Northwestern University, 2145 Sheridan Road, A335, Evanston, IL 60208-3109,
Phone: (847) 491-4008, Fax: (847) 491-4011, Email: pdc@northwestern.edu

and

Pattharin Sarutipand
Ph.D. Candidate, Department of Civil and Environmental Engineering,
Northwestern University, 2145 Sheridan Road, A316, Evanston, IL 60208-3109,
Phone: (847) 467-1852, Fax: (847) 491-4011, Email: p-sarutipand@northwestern.edu
Abstract

We present a quadratic programming formulation for the problem of obtaining optimal maintenance and repair policies for multi-facility transportation infrastructure systems. The proposed model provides a computationally-tractable framework to support decision-making, while accounting for economic interdependencies that link the facilities that comprise these systems. To demonstrate the advantages of the proposed model, we present numerical examples that capture economic interdependencies reflecting both costs associated with disruptions/loss of throughput, as well as the benefits associated with coordinating intervention schedules of adjacent facilities to reduce costs associated with resource and personnel delivery. The results illustrate situations where it is optimal to coordinate (synchronize or alternate) interventions for clusters of facilities in transportation systems.


**Introduction**

Transportation infrastructure management refers to the process of making decisions concerning the allocation of resources for maintenance and repair (M&R) of the facilities that comprise transportation systems. The objective in making these decisions is to ensure that the systems are capable of performing the functions for which they were designed and built, while accounting for limited resource availability. Management decisions trade off user costs, which correspond to a fraction of the costs associated with travel time, fuel consumption, vehicle depreciation and maintenance, with agency costs. As facilities deteriorate due to utilization and weathering, the rate at which user costs accrue increases. Agency costs are incurred to improve condition, and thus, reverse the effects of deterioration.

The motivation for the work presented herein is that existing optimization models to support management decisions sacrifice important functional characteristics in favor of computational tractability/simplicity. In particular, existing models rely on the assumption that transportation systems consist of facilities that are homogeneous (with respect to costs and deterioration). Furthermore, they do not include information to identify the individual components of a transportation system, i.e., facilities are assumed to be identical. Optimal M&R policies obtained with such models specify the same set (probability distribution) of decisions for all facilities that are in a given state, i.e., facilities are classified by their condition. This type of model is attractive because the computational effort to find optimal policies is independent of the number of facilities that comprise the system. However, omitting information that identifies the individual facilities makes it impossible to capture economic relationships that link them. This, in turn, constitutes an obstacle to providing effective decision support because significant benefits and costs in the management process can be directly attributed to interdependencies that link a system's facilities. Costs related to congestion, for example, fall under this category because congestion is often triggered by disruptions in functionally-related components of a transportation system.

The work presented herein constitutes an important step to address the limitations described in the preceding paragraph. Specifically, we formulate the problem of obtaining optimal M&R policies for multi-facility transportation systems as a quadratic program (i.e., a QP problem). The model provides a computationally-tractable approach to obtain optimal M&R policies while capturing costs and constraints that link the facilities that comprise the system. In the formulation, each facility's deterioration process is identified and represented as a linear system, i.e., an autoregressive moving average model with exogenous inputs (ARMAX) model. The quadratic objective is used to capture (pair-wise) economic dependencies, which may, for example, reflect costs associated with disruptions/loss of throughput or benefits associated with coordinating M&R schedules for adjacent facilities to reduce costs associated with resource and personnel delivery.

**Motivation for Capturing Interdependencies: An Example**

Existing optimization models for the problem of finding M&R policies for multi-facility transportation systems are formulated as constrained Markov Decision Processes (MDPs) using the notion of “randomized policies” and are solved as linear programs (Golabi et al., 1982; Golabi and Shepard, 1997; Murakami and Turnquist, 1997; Smilowitz and Madanat, 2000). In
this type of model, individual facilities are not identified and are assumed to be homogeneous. As a result, optimal policies specify the same set (probability distribution) of M&R actions for all facilities that are in a given state, i.e., they are classified by their condition. Linear constraints can be included in these models to impose restrictions that apply to the system, e.g., budget that can be allocated to M&R actions, level of service requirements, etc. These models are attractive because the computational effort to find optimal M&R policies is independent of the number of facilities that comprise the system. Unfortunately, omitting information about the individual facilities makes it impossible to capture interdependencies that can play an important role in the management process, as is the case in the following example.

![Figure 1 Hypothetical Transportation Network](image)

Figure 1 represents a hypothetical transportation network with two paths between node 1 (origin) and node 4 (destination). We assume that the four links in the network are homogeneous with respect to their deterioration.

The fact that existing models classify facilities by their condition means that if links (1,2) and (1,3) are in the same condition, an optimal policy would specify the same M&R action for both of them. Also, the homogeneous cost structure means that it costs exactly twice as much to apply an action to both links as it does to apply the action to either link individually. In practice, however, one expects M&R policies to specify that (major) M&R actions on links (1,2) and (1,3) be performed at different times in order to minimize the disruption to the system. Such policies would reflect a cost structure in which disrupting the two links simultaneously costs much more than twice the cost of disrupting each link individually. Links (1,2) and (1,3) are said to exhibit a substitutable relationship. For analogous reasons, one also expects that M&R actions on links (1,2) and (2,4), or on links (1,3) and (3,4), would be synchronized. The links in each pair are said to exhibit a complementary relationship.

The remainder of the paper presents a model that we propose to account for the interdependencies, such as the ones in the previous example. We also consider numerical examples that are meant to illustrate the importance of capturing interdependencies in the management of infrastructure systems. We begin, however, by providing an overview of the relevant literature.

**Literature Review**

Multi-component M&R optimization models have been developed from multiple perspectives to address numerous applications. Thomas (1986), Cho and Parlar (1991) and Dekker et al. (1997) present comprehensive reviews of the literature. Thomas (1986) and Dekker et al. (1997) classify the models based on the possible interdependencies between the facilities that comprise a system, which can be either structural/functional, stochastic or economic. Functional dependence refers
to situations where the successful operation of a system requires the successful operation of a minimum number of different types of components, i.e., the reliability of the system depends on the reliability of its components. In the context of M&R, system (or component) failures present opportunities to inspect, maintain, repair or replace both failed and functional components. Stochastic dependence refers to situations where the time-to-failure distributions of different components are dependent (either due to interactions or because they are influenced by similar factors, e.g., environment or loading). This means that knowledge about the condition (or failure) of a subset of the components provides an opportunity to update the time-to-failure distributions and the M&R policy for other components. Finally, economic dependence refers to situations where the components of a system are linked by resource constraints or level of service requirements, or by the system-wide cost-benefit structure, i.e., the total cost of applying actions on a set of components can be different than the sum of the individual component costs. As an example of economically dependent components, consider a system in which the maintenance of each component requires preparatory or set-up work that can be shared when several components are maintained simultaneously.

It is clear that real-world systems, including transportation systems, simultaneously exhibit the three types of interdependencies described above. For simplicity, however, models tend to focus on the single type of dependence that dominates the behavior of the system under consideration or that is most relevant to the decision-making situation. In the context of developing M&R policies for multi-facility transportation systems, interdependencies derive from the spatial distribution of links within the system, as well as the functional characteristics of the system, i.e., network topology/connectivity and traffic flow patterns. These relationships have economic implications, and thus, we focus on developing joint, system-wide M&R policies for multi-facility transportation systems that account for economic interdependencies. For example, we seek to obtain M&R policies that account for the benefits and costs that stem from the complementary or substitutable relationships described in the example presented in the previous section, or for the benefits associated with coordinating M&R schedules for adjacent facilities to reduce set-up costs associated with resource and personnel delivery. The remainder of the section provides an overview of related work, focusing on limitations that motivate and justify the proposed approach.

Recognizing that functional relationships may dominate the M&R decision-making process, a number of studies have addressed the problem of sequencing and scheduling M&R activities/work zones with the objective to minimize the disruption, i.e., incremental system-wide travel time/costs, on a network. The resulting optimization models are typically formulated as integer programs and solved using techniques such as genetic algorithms (Fwa and Tan, 1996; Fwa and Muntasir, 1998), Tabu Search (Chang et al., 2001), or simulation (Sanford-Bernhardt and McNeil, 2004). In contrast to the constrained MDP approach, these models emphasize the spatial elements of the problem over the temporal ones, i.e., the focus is to schedule/sequence a set of (preetermined) activities during a construction season or over a short planning horizon.

Other studies in the context of managing transportation systems have recognized that interdependencies can be significant in M&R decision-making. For example, Martland et al. (1990) proposes a simple “rule-based” approach to take advantages of complementarities in rail replacement decision-making. Similarly, Hajdin and Lindenmann (2005) describe the notion of corridors and present a network optimization model that yields optimal intervention bundles on complementary sections in a road network. Synchronizing interventions can also have negative
impacts on performance. These are discussed in some detail by Adey et al. (2003) in the context of bridge management. Generally, the aforementioned studies rely on economic analysis to evaluate different M&R scenarios. Functional or economic dependencies are used to cluster the system's components, thereby limiting the number of scenarios under consideration. Philosophically, these approaches provide a better balance of the spatial and temporal tradeoffs that exist in M&R decision-making. Unfortunately, the computational effort to carry out these types of analysis makes them impractical, and in turn, motivates the need for a systematic and efficient framework.

The wide use and acceptance of finite (state and action) MDP formulations for periodic review M&R optimization problems means that perhaps the most natural way to develop a framework for systems of interdependent facilities is to consider multi-dimensional MDP formulations for the problem. In the management science literature, these types of models are referred to as Parallel Machine/Equipment Replacement Problems (Vander Veen, 1985; Jones et al. 1991; McClurg and Chand, 2002; Chen, 1998; Childress and Durango-Cohen, 2005). These formulations provide an attractive framework to model economic interdependencies because the state and action space, as well as the deterioration process of each facility in the system is fully identified. In addition, there is a great deal of flexibility in terms of specifying M&R costs. Unfortunately, the fact that the state and action spaces are discrete, leads to significant computational problems in solving and analyzing such models. These difficulties are well-known and referred to as the “curse of dimensionality”. The cause of these problems is that solution approaches for MDPs require enumerating all possible interventions for every system state and for every decision-making stage. Because the state and action spaces are discrete, the total numbers of possible system states and interventions increase exponentially with the number of facilities in the system. This consideration makes the approach impractical, even for transportation systems with small numbers of facilities.

The computational difficulties associated with multi-dimensional MDP formulations have motivated the development of other approaches to address M&R optimization problems for systems of interdependent facilities. Comprehensive reviews are presented in Thomas (1986), Cho and Parlar (1991), and Dekker et al. (1997). One of the most common approaches to address the problem involves adapting methods that have been developed in the context of coordinating inventory replenishment policies for multiple products. Essentially, the approach involves two steps. First, facilities are divided into groups depending on the costs or benefits associated with coordinating M&R interactions. Usually facilities that share set-up costs are clustered. The second step is to obtain optimal M&R policies for each of the clusters. A lot of work has been done in terms of characterizing the structure of optimal M&R policies for highly specific problem instances. Often, these results rely on assumptions such as homogeneous facilities, predetermined clusters of facilities (as opposed to optimal ones), and simplified cost structures that can be unattractive for the analysis of transportation systems. In particular, few to none of the models in the literature are capable of capturing the costs and benefits stemming from the substitutable relationship illustrated in the example presented in the previous section.

---

1 Childress and Durango-Cohen (2005), for example, show how the solution to a modest, single-stage PMRP with 15 facilities, 5 condition-states and 2 actions for each facility, requires solving a linear program with over 15 thousand variables and over 77 million constraints. This consideration is important given the size of transportation systems. Golabi and Pereira (2003), for example, recently developed a pavement management system for a road network with over 40 thousand sections.
In this paper, we build on an optimal control framework that is often used to address the curse of dimensionality. In particular, the M&R optimization model proposed in this paper is an extension of the framework presented in Durango-Cohen and Tadepalli (2006) for infrastructure facilities. The aforementioned work presents a discrete-time, linear, stochastic optimal model that is capable of processing multidimensional arrays of condition data and subsequently exploiting this information to support M&R decisions. The authors show that in cases where the cost functions can be represented (or approximated) by second order polynomials the problem corresponds to a problem with linear dynamics and a quadratic criterion. Significantly, the paper shows that the computational effort to obtain and implement optimal policies increases polynomially with the number of variables in the model, i.e., the framework effectively overcomes the computational difficulties associated with the multi-dimensional MDP approach as described above. This, in turn, motivates and justifies the extension of the framework to address the problem of a system comprised of multiple facilities.

As a matter of background and to contrast the framework presented in Durango-Cohen and Tadepalli (2006), we note that optimal control theory has been used to address M&R optimization of infrastructure facilities (cf. Friesz and Fernandez (1979); Markow and Balta (1985); Tsunokawa and Schofer (1994); Li and Madanat (2002)). These models are usually formulated as continuous-time problems and have been used to derive analytical properties. Friesz and Fernandez (1979) is of particular interest as it is, to the best of our knowledge, the first M&R optimization models proposed for transportation infrastructure. Unfortunately, the attractive computational features of the framework presented by Durango-Cohen and Tadepalli (2006), typically do not extend to the aforementioned, continuous-time control models (without imposing further structure on the problem's parameters).

Model Formulation

We begin this section by introducing notation and by describing the cost components that we use to specify our model. We then present a QP formulation to obtain optimal M&R policies for systems of economically-interdependent facilities.

Notation and Parameter Specification

We consider the problem of managing, i.e., finding an optimal M&R policy for a transportation system that consists of \( n \) facilities over a planning horizon consisting of \( T \) periods/stages. The state and decision vectors for the problem are defined as follows:

\[ X_t \in \mathbb{R}^n : \text{Vector that represents the system's condition at the start of stage } t. \quad X_t \equiv [x_1^t, x_2^t, \ldots, x_n^t] \]

Each vector component represents the condition of a given facility, where larger values of a vector component correspond to worse condition.

\[ Y_t \in \mathbb{R}^n : \text{Decision vector representing the set of M&R decisions for period } t. \quad Y_t \equiv [y_1^t, y_2^t, \ldots, y_n^t] \]

Each vector component corresponds to the investment level/maintenance rate for the corresponding facility.

The fundamental assumptions that we use to specify our model are as follows:

---

2 The symbol ' is used to denote the transpose of a vector.
A1: The period cost function, $p()$, and the salvage/residual value function, $s()$, can be represented (or approximated) by general second-order polynomials. Using linear algebra notation, we may write the polynomials as follows:

$$p(X_t, Y_t) = X_t'AX_t + X_t'BY_t + Y_t'CY_t + d'X_t + e'Y_t + f$$

$$s(X_{T-1}) = X_{T-1}'LX_{T-1} + w'X_{T-1} + z$$

A2: Facility deterioration is represented as a deterministic linear system of the form:

$$X_{t+1} = GX_t + HY_t + K$$

where G, H and K, respectively, are matrices and a vector of appropriate dimensions.

Prior to continuing with the model's description, we note that:

• In terms of understanding the model, we note that continuous decision variables can be interpreted as maintenance rate or as an investment level. This interpretation is consistent with earlier models in the field (cf. Friesz and Fernandez (1979)) and is appropriate for tactical and strategic models, which might, for example, be used for budgeting.

• The quadratic cost structure is not overly restrictive because, for example, it is possible to obtain optimal M&R policies for general classes of continuous cost functions by solving a sequence of problems. In each problem the cost function is approximated by a second-order Taylor Series expanded about a different point. The procedure is analogous to the Newton-Raphson method for solving systems of equations/optimization problems and is discussed further in Dreyfus (1977).

• Equation (3) corresponds to an AutoRegressive Moving Average with eXogenous input (ARMAX) model. ARMAX models provide a convenient, flexible and rigorous framework to formulate and estimate infrastructure deterioration models; however, the assumption that they are appropriate requires (empirical) validation. While preliminary statistical analysis and results are encouraging (Chu and Durango-Cohen, 2006), the assumptions embedded in ARMAX models may not be universally valid, e.g., that the effect of M&R actions is linear and additive. Even though there is flexibility to (partially) address some of the limitations, it is important to realize that the ARMAX framework may be inadequate in certain situations. Thus, the use of ARMAX models should be interpreted as analogous to the use of linear programming, even though most realistic problems probably do not exhibit the linear structure that is assumed.

• Our primary objective in this paper is to develop a framework that captures the effect of economic interdependencies and of heterogeneity on M&R policies. To simplify the presentation, we assume that facility deterioration is deterministic, i.e., the vector K is constant. This assumption reduces the generality of the framework, but leads to an optimization problem that is both flexible enough to capture interdependencies and heterogeneity, and that is practical for systems with large numbers of facilities. In contrast, the constrained MDP is practical but inflexible, and the multi-dimensional MDP is flexible but impractical. We also think it is appropriate to raise the following points:

---

3 State-space specifications of ARMAX models are used to formulate and estimate deterioration and measurement-error models for transportation infrastructure. The models are estimated using deflection and pressure measurements generated by sensors embedded in an asphalt pavement.
- The assumption of deterministic deterioration has been used in previous models in the literature;
- Recent statistical studies such as Archilla and Madanat (2000) and Prozzi and Madanat (2002) have shown that a large fraction of the variability in infrastructure deterioration should be attributed to (unobserved) heterogeneity among facilities, as opposed to aleatory uncertainty. Constrained MDPs, unnecessarily, attribute all of the variability to aleatory uncertainty. The proposed model (unnecessarily) attributes all of the variability to heterogeneity. Clearly, both are extremes and the ideal model lies somewhere in between; and
- The solution to optimization of problems whose dynamics are governed by stochastic linear systems and whose objective is quadratic, i.e., stochastic linear-quadratic regulators, is discussed in references such as Bertsekas (1995). Using this general framework for M&R optimization would seem to be an important direction for research.

To understand how the parameters for the model are specified, we proceed to discuss the cost components that can be captured in the model.

**Condition-Dependent User Costs:** These costs correspond to the sum of user costs that are directly attributable to a facility's condition. In terms of specifying the period cost function, Equation (1), we assume that these costs are independent for each of the facilities that comprise the system. Thus, in stage $t$ the costs may be written as:

$$CDUC = \sum_{i=1}^{n} \left[ a_i (x_i^t)^2 + d_i^t + f_i^{cduc} \right]$$

(4)

Intuitively, one expects the condition-dependent costs for each facility to be convex and increase as condition worsens. Thus, $a_i > 0$ and $d_i > 0$, $\forall i$. $f_i^{cduc}$ is the portion of condition-dependent user costs attributed to facility $i$ that is fixed and thus included in the constant term $f$.

**Independent User and Agency Costs Associated with M&R Decisions:** These costs correspond to the sum of costs associated with M&R of the facilities that comprise the system and that are independent of all other facilities. Thus, in stage $t$ the costs may be written as

$$IM&RC = \sum_{i=1}^{n} \left[ c_i (y_i^t)^2 + b_i y_i^t + e_i y_i^t + f_i^{IM&RC} \right]$$

(5)

Intuitively, one expects that M&R costs for each facility would increase with the level of intensity of M&R actions. Thus, $c_i > 0$ and $e_i > 0$, $\forall i$. One might also expect that the application of an action would be either independent or increasing with worsening conditions. Thus, $b_i \geq 0$. $f_i^{IM&RC}$ is the portion of the costs attributed to $i$ that is fixed, i.e., included in the constant term $f$.

**Dependent User and Agency Costs Associated with M&R Decisions:** These costs correspond to the sum of costs associated with M&R decisions that link or that are dependent on the state or on the decisions for other facilities in the system. In stage $t$ these costs may be written as
The first set of terms reflects how the costs of investing \( y_j^i \) on \( j \) depend on the condition of facility \( i \), \( x_i^t \). One expects these set of terms to be nondecreasing with both \( x_i^t \) and \( y_j^i \). Therefore, \( b_{ij} \geq 0 \). The second set of terms captures the link between the decisions for the different facilities in the system. When \( c_{ij} > 0 \) the terms might reflect costs associated with loss of functionality/throughput when facilities \( i \) and \( j \) are disrupted simultaneously. When \( c_{ij} < 0 \) the terms might reflect savings derived from personnel and equipment delivery costs when adjacent facilities are maintained in the same period. Finally, the last set of terms accounts for fixed costs. Without loss of generality, we also assume that all parameters are stationary.

**Salvage Value:** We assume that the salvage/residual value of a facility is independent of other facilities, and thus, we assume that matrix \( L \) is diagonal.

**Deterioration Process:** Without loss of generality, we assume that the deterioration of a facility is independent of the condition of and maintenance applied to other facilities, and that the models only require first-order lag variables. Thus, \( G \) and \( H \) are \( n \times n \) diagonal matrices.

### Mathematical Model

The problem of obtaining an optimal M&R policy can be written as follows:

\[
\text{Minimize: } \sum_{t=1}^{T} \delta^{t-1} \left[ X_t^i AX_t + X_t^i BY_t + Y_t^i CY_t + dX_t + eY_t + f \right] + \delta^T \left[ X_{t+1}^i LX_{t+1} + wX_{t+1} + z \right]
\]

Subject to:

\[
X_{t+1} = gX_t + hY_t + k \quad \text{(8)}
\]

\[
X_1 = \hat{X}_1 \quad \text{(9)}
\]

\[
Y_t \geq 0 \quad \text{(10)}
\]

Equation (7) represents the minimum discounted cost over \( T \) stages. Equation sets (8) and (9) represent the deterioration of the facilities that comprise the system and their initial condition, respectively. Equation (10) constrains the investment level in each period to be nonnegative. We assume that the inequality applies componentwise to the vector \( Y_t \).

### Numerical Examples

In this section, we introduce examples where we compare optimal policies obtained with the proposed model to policies obtained with a model that ignores interdependencies between the facilities that comprise a transportation system. The latter type of policy is intended to mimic policies obtained when using the constrained MDP framework. Our objective is to show that the proposed model has appealing features that elude existing M&R optimization models for transportation infrastructure. The reader interested in computational issues associated with
solving (large-scale) quadratic programs is referred to Gil et al. (1981). The examples are presented in order of increasing complexity.

We solved the QPs to obtain the numerical results with a robust nonlinear optimization solver, LOQO. This solver is publicly available online through the NEOS servers located at Northwestern University and the Argonne National Laboratory. Further information is available through http://www-neos.mcs.anl.gov/neos/solvers/nco:LOQO/AMPL.html.

**Basic Complementary and Substitutable Networks**

We begin by considering two simple transportation systems comprised of two links and one origin-destination pair. The two types of networks: substitutable and complementary are presented in Figures 2(a) and 2(b), respectively.

As discussed earlier, existing optimization models ignore interdependencies and thus would specify the same M&R policy for the two networks. Intuitively, one expects the application of M&R actions on the substitutable network to be performed at different times in order to avoid disrupting all available paths between the origin and destination. In our model, the economic impact of this disruption is captured with a positive cost parameter $c_{12}$. Additionally, the user costs that derive from the application of M&R decisions can be influenced by the condition of substitutable links because a fraction of the users are likely to divert in order to avoid disruptions. These costs can be captured in our model through parameter $b_{12}$. In the complementary network, on the other hand, one expects the application of M&R actions on this network to be synchronized and performed during the same stage. This would reduce loss of throughput/disruption caused when either link 1 or 2 is maintained. For the agency, synchronization of M&R activities on sequential links may lead to some savings in costs associated with resource and personnel delivery. In contrast to the substitutable network, parameter $c_{12}$ would be negative, representing the agency cost reduction.

![Type of Network](image)

**Figure 2: Types of Network**

**Condition-Dependent Policy: Base Case**

As a baseline for the more complicated case, we examine the results of the case when interdependencies are ignored. Thus, the results are independent of types of network. Table 1 shows the parameters of Equations (7) and (8) that are used in the example.

<table>
<thead>
<tr>
<th>Link</th>
<th>$a_i$</th>
<th>$b_{ij}$</th>
<th>$c_{ij}$</th>
<th>$d_i$</th>
<th>$e_i$</th>
<th>$f$</th>
<th>$g_i$</th>
<th>$h_i$</th>
<th>$k_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$b_{12}$</td>
<td>$c_{12}$</td>
<td>-50</td>
<td>0</td>
<td>1049.5</td>
<td>1</td>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$b_{22}$</td>
<td>-50</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
Prior to presenting the results of the numerical study, we emphasize that our objective is to develop qualitative insights about how parametric changes affect the optimal policies of managing facilities. Thus, the parameters used in the study are not representative of any particular facility, although the situation considered was “inspired” by a pavement management situation. To simplify the interpretation of the numerical results we consider an infinite horizon version of the problem. This approximation is widely accepted in the management of transportation infrastructure where planning horizons tend to be long and uncertain. A nice feature of infinite horizon problems is that suitable parameter choices lead to optimal policies that converge to a steady-state, i.e., the policy prescribes periodic interventions for the links in the system. Thus, optimal steady-state policies provide a convenient way to contrast the different models that we consider. The other parameters in the model were also chosen to simplify the interpretation of the results. In particular, the cost parameters were chosen so that the infinite horizon sum of discounted costs of managing each facility independently would be $0. The discount rate was set to 5%. The deterioration was chosen so that it would take 10 periods to deteriorate through a hypothetical scale with range 100 - intended to mimic the PCI scale.

The results for the base case where the parameter $b_{12}$ and $c_{12}$ are set to zero to mimic the case where interdependencies are ignored show that the optimal policy is to apply the same amount of investment on both links, as illustrated in Figure 3.

![Figure 3: Optimal Investment level of the Base Case](image)

The result indicates that, after the system reaches that steady state, the same actions apply to the same link condition; showing that the QP model to obtain the condition-dependent policy is intuitively similar to existing models. In the following section, we show how the additional flexibility/parameters in the QP model can be used to capture heterogeneity and interdependencies, and thus obtain a more general facility-condition-dependent policy. This additional flexibility is exploited to develop a more appealing model of the trade-offs between user and agency costs.

**Capturing Interdependencies of the Substitutable Network**

In this section, we discuss a parametric study where we isolate the effect of parameters $b_{12}$ and $c_{12}$ and consider their effect on the ensuing optimal M&R policies. We begin by considering the traffic disruption cost that caused by simultaneous M&R actions. As mentioned above, this cost is represented by a positive $c_{12}$.

The following qualitative observations follow from varying $c_{12}$: When $c_{12}$ is small, the optimal policies are the same as in the base case. The policies are to apply the same investment on both links at every year. This result presents the case where the benefits of maintaining the facility outweighs the traffic disruption caused by substitutability of link 1 and 2. When $c_{12}$ is large, on the other hand, the optimal policies differ from the base case. The optimal steady-state policy specifies alternating investments between link 1 and 2, as shown in Figure 4; suggesting
that the traffic disruption dominates the decision-making process. We notice that the investment magnitude in the later case is twice as much of the former case. When traffic disruptions are not accounted for, a moderate maintenance rate is preferable to a severe one, and should, therefore, reduce the sum of expected discounted costs. However, when the disruption costs are substantially high, the policies are adjusted to reflect the trade-offs between the instantaneous cost of disrupting users and the sum of expected discounted costs. Therefore, an alternate policy is recommended for this situation.

Next, we consider user costs associated with rerouting when the M&R activity is applied to a functionally-related link. The rerouting cost is represented by a positive $b_{12}$, as discussed earlier. By varying parameter $b_{12}$, we observe that: When $b_{12}$ is small, the optimal policies are the same as in the base case. The policies are to apply the same investment on both links in every period. When $b_{12}$ is large, the optimal policies differ from the base case. The optimal steady-state policies now is to alternate the investment between link 1 and 2, also as is shown in Figure 4; suggesting that the costs induced by rerouting dominates the decision making and justifies the optimal policy.

The cost induced by rerouting depends on the investment of one link and the condition of the alternate link. Intuitively, links are maintained when their condition is poor. There is an incentive to alternate interventions on substitutable links in order to maintain alternative routes in good condition. When the users' inconvenience caused by the rerouting is marginal, maintaining link when the alternate link is in just about good, but not necessarily very good, condition does not magnify the users' inconvenience. Therefore the application of the investment on both links, when they are in the same condition, justifies the optimal policy. This is the situation when $b_{12}$ is small. Moreover, when the rerouting impact $b_{12}$ increases, the link condition at the steady-state decreases, i.e., steady-state condition is better. This arises so that the alternate link is deliberately counted as a potential-alternate route.

When the rerouting impact of M&R application on each link is significantly high, the application of the investment on both links may no longer optimal. One expects the alternative route to be in a very good condition. As a result, alternating the link condition justifies the optimal policy. For example, before link 1 will be maintained, link 2, which is an alternative route, should be in very good condition. So that link 2 will be ready to be use as an alternative route during the period that link 1 is maintained. In our numerical example, the condition of each link varies alternately between two stable states, e.g., condition index of link 2 is alternated between 7.5 and 17.5. To achieve this situation, the investment must be alternated between links as well. As shown in Figure 4, before the investment is applied on link 1 in period $t+1$, the condition of link 2 is improved, by application of investment, beforehand in period $t$.

Capturing Functional Interdependencies of the Complementary Network
In this section the functional interdependencies of the complementary network are captured by varying the corresponding parameter, $c_{12}$. As mentioned earlier, the savings in M&R expenditures are represented by the negative $c_{12}$. We limit the lower bound of $c_{12}$ to -1 because the savings from coordinating maintenance on both links should not exceed the cost of maintaining an individual link.

The result shows that the optimal policy does not change from the base case. This arises because when one of the complementary links is maintained, a bottleneck is already created along the path. As a result, the condition or level of service beyond the maintained link is not as good as the normal condition, but it will be the same as the level of service on the maintained link. Therefore, the impact, or namely loss of throughput, of maintaining the complementary links together would be the same as doing it on either one of them. Thus, there is an incentive to coordinate the maintenance schedule of the complementary network, which is opposite to the substitutable network.

**Conclusions**

We present a quadratic programming formulation for M&R optimization of multi-facility transportation systems. The proposed model provides a computationally-tractable framework that can be used to capture important economic and functional interdependencies that link facilities in transportation systems. The model itself constitutes an extension of the discrete-time optimal control model of Durango-Cohen and Tadepalli (2006) for single facilities. The work described herein is motivated both by the inability of existing models (formulated as constrained MDPs) to capture the aforementioned interdependencies, and by the computational limitations of other approaches that have been suggested for the problem (multi-dimensional MDPs or continuous-time optimal control theory).

The advantages of the proposed model are demonstrated through a set of examples where the quadratic criterion is used to represent costs associated with (i) loss of throughput/traffic disruptions caused by the presence of simultaneous M&R activities in a transportation system, (ii) diverting traffic/rerouting users while a link is being maintained, and (iii) resource and personnel delivery (and the corresponding savings when adjacent facilities are maintained simultaneously). The examples are used to verify the intuitive result that the maintenance on complementary links should be performed simultaneously, while the maintenance on substitutable links should alternate between periods. Overall, the results highlight the importance of interdependencies in M&R decision-making and suggest the need for further research in this area.

**References**


