Modeling Consumers’ Choice of Multiple Items Simultaneously: A Methodological Approach with an Application to Vehicle Holdings and Use

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Overview

- Introduction
- Functional form of utility function
- Stochastic form of utility function
- Specific model structures
- Empirical illustration
- Conclusions
Several consumer demand choices are characterized by multiple discreteness

- Vehicle type holdings and usage
- Activity type choice and duration of participation
- Airline fleet mix and usage
- Carrier choice and transaction level
- Brand choice and purchase quantity
- Stock choice and investment amount
Multiple discreteness

- Choice of multiple alternatives simultaneously

Modeling methodologies of multiple discrete situations

- Traditional random utility-based (RUM) single discrete choice models
  - Number of composite alternatives explodes with the number of elemental alternatives

- Multivariate probit (logit) methods
  - Not based on a rigorous underlying utility-maximizing framework of multiple discreteness

Other issues with these methods

- Cannot accommodate the diminishing marginal returns (i.e., satiation) in the consumption of an alternative
- Cumbersome to include a continuous dimension of choice
Modeling methodologies of multiple discrete situations

- Two alternative methods proposed by Wales and Woodland (1983)
  - Amemiya-Tobin approach
  - Kuhn-Tucker approach

- Both approaches assume a direct utility function $U(x)$ that is assumed to be quasi-concave, increasing, and continuously differentiable with respect to the consumption quantity vector $x$

- Approaches differ in how stochasticity, non-negativity of consumption, and corner solutions (*i.e.*, zero consumption of some goods) are accommodated
Methods proposed by Wales and Woodland

Amemiya-Tobin approach
- Extension of the classic microeconomic approach of adding normally distributed stochastic terms to the budget-constrained utility-maximizing share equations
- Direct utility function $U(x)$ assumed to be deterministic by the analyst, and stochasticity is introduced post-utility maximization

Kuhn-Tucker (KT) approach
- Based on the Kuhn Tucker or KT (1951) first-order conditions for constrained random utility maximization
- Employs a direct stochastic specification by assuming the utility function $U(x)$ to be random (from the analyst’s perspective) over the population
- Derives the consumption vector for the random utility specification subject to the linear budget constraint by using the KT conditions for constrained optimization
- Stochastic nature of the consumption vector in the KT approach is based fundamentally on the stochastic nature of the utility function
Advantages of KT approach

- Constitutes a more theoretically unified and consistent framework for dealing with multiple discreteness consumption patterns
- Satisfies all the restrictions of utility theory
- Stochastic KT first-order conditions provide the basis for deriving the probabilities for each possible combination of corner solutions (zero consumption) for some goods and interior solutions (strictly positive consumption) for other goods
- Accommodates for the singularity imposed by the “adding-up” constraint

Problems with KT approach used by Wade and Woodland

- Random utility distribution assumptions lead to a complicated likelihood function that entails multi-dimensional integration
Studies that used the KT approach for multiple discreteness

Kim et al. (2002)
- Used the GHK simulator to evaluate the multivariate normal integral appearing in the likelihood function in the KT approach
- Used a generalized variant of the well-known translated constant elasticity of substitution (CES) direct utility function
- Not realistic for practical applications and is unnecessarily complicated

Bhat (2005)
- Introduced a simple and parsimonious econometric approach to handle multiple discreteness
- Based on the generalized variant of the translated CES utility function but with a multiplicative log-extreme value error term
- Labeled as the multiple discrete-continuous extreme value (MDCEV) model
- MDCEV model represents the multinomial logit (MNL) form-equivalent for multiple discrete-continuous choice analysis and collapses exactly to the MNL in the case that each (and every) decision-maker chooses only one alternative

Several studies in the environmental economics field

Phaneuf et al., 2000; von Haefen et al., 2004; von Haefen, 2003a; von Haefen, 2004; von Haefen and Phaneuf, 2005; Phaneuf and Smith, 2005
- Used variants of the linear expenditure system (LES) and the translated CES for the utility functions, and used multiplicative log-extreme value errors
Functional form of utility function

\[ U(x) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left[ \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right] \]

- \( U(x) \) is a quasi-concave, increasing, and continuously differentiable function with respect to the consumption quantity vector \( x \)
- \( \psi_k, \gamma_k \) and \( \alpha_k \) are parameters associated with good \( k \)
Assumptions

- Additive separability
  - All the goods are strictly Hicksian substitutes
  - Marginal utility with respect to any good is independent of the level of consumption of other goods

- Weak complementarity
Role of $\psi_k$

$$
\frac{\partial U(x)}{\partial x_k} = \psi_k \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k^{-1}}
$$

- $\psi_k$: baseline (at zero consumption) marginal utility
- $\frac{\psi_k}{\psi_l}$: marginal rate of substitution at zero consumption
- Higher baseline $\psi_k$ implies less likelihood of a corner solution for good $k$
Role of
Indifference Curves

Indifference Curves Corresponding to Different Values of $\gamma_1$

$\psi_1 = \psi_2 = 1$
$\alpha_1 = \alpha_2 = 0.5$
$\gamma_2 = 1$
Role of $\gamma_k$

Effect of $\gamma_k$ Value on Good $k$’s Subutility Function Profile
Role of $\alpha_k$

Effect of $\alpha_k$ Value on Good $k$'s Subutility Function Profile
Empirical identification issues associated with utility form

- Combination profile ($\alpha = 0.2, \gamma = 30$)
- $\gamma_k$ profile ($\alpha^* = 0, \gamma^*_k = 45$)
- $\alpha_k$ profile ($\alpha^{**} = 0.745, \gamma^{**}_k = 1$)

$\psi_k = 1$ for all profiles

Alternative Profiles for Moderate Satiation Effects with Low $\alpha_k$ Value and High $\gamma_k$ Value
Empirical identification issues associated with utility form

$\psi_k = 1$ for all profiles

Alternative Profiles for Moderate Satiation Effects with High $\alpha_k$ Value and Low $\gamma_k$ Value
Empirical identification issues associated with utility form

Alternative Profiles for Low Satiation Effects with High $\alpha_k$ Value and High $\gamma_k$ Value
Empirical identification issues associated with utility form

Alternative Profiles for High Satiation Effects with Low $\alpha_k$ Value and Low $\gamma_k$ Value
Stochastic form of utility function

- Overall random utility function

\[ U(x) = \sum_k \frac{\gamma_k}{\alpha_k} \left[ \exp(\beta'z_k + \varepsilon_k) \right] \cdot \left\{ \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\} \]

- Random utility function for optimal expenditure allocations

\[ U(x) = \sum_k \frac{\gamma_k}{\alpha_k} \left[ \exp(\beta'z_k + \varepsilon_k) \right] \cdot \left\{ \left( \frac{e_k}{\gamma_k p_k} + 1 \right)^{\alpha_k} - 1 \right\} \]
Lagrangian and KT Conditions

\[ L = \sum_{k} \frac{\gamma_k}{\alpha_k} \left[ \exp(\beta' z_k + \varepsilon_k) \right] \left\{ \left( \frac{e_k}{\gamma_k p_k} + 1 \right)^{\alpha_k} - 1 \right\} - \lambda \left[ \sum_{k=1}^{K} e_k - E \right] \]

\[ \left[ \frac{\exp(\beta' z_k + \varepsilon_k)}{p_k} \right] \left( \frac{e_k^*}{\gamma_k p_k} + 1 \right)^{\alpha_k - 1} \]

\[ \left[ \frac{\exp(\beta' z_k + \varepsilon_k)}{p_k} \right] \left( \frac{e_k^*}{\gamma_k p_k} + 1 \right)^{\alpha_k - 1} \]

\[ \lambda = \frac{\exp(\beta' z_1 + \varepsilon_1)}{p_1} \left( \frac{e_1^*}{\gamma_1 p_1} + 1 \right)^{\alpha_1 - 1} \]
KT conditions

\[ V_k + \varepsilon_k = V_1 + \varepsilon_1 \quad \text{if } e_k^* > 0 \quad (k = 2, 3, \ldots, K) \]

\[ V_k + \varepsilon_k < V_1 + \varepsilon_1 \quad \text{if } e_k^* = 0 \quad (k = 2, 3, \ldots, K), \text{ where} \]

\[ V_k = \beta'z_k + (\alpha_k - 1)\ln\left(\frac{e_k^*}{\gamma_k p_k} + 1\right) - \ln p_k \quad (k = 1, 2, 3, \ldots, K) \]
**General econometric model structure and identification**

\[ P(e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0) = |J| \int_{\varepsilon_1=-\infty}^{+\infty} \int_{\varepsilon_{M+1}=-\infty}^{+\infty} \int_{\varepsilon_{M+2}=-\infty}^{+\infty} \cdots \int_{\varepsilon_{K-1}=-\infty}^{+\infty} \int_{\varepsilon_{K}=-\infty}^{+\infty} f(\varepsilon_1, V_1 - V_2 + \varepsilon_1, V_1 - V_3 + \varepsilon_1, \ldots, V_1 - V_M + \varepsilon_1, \varepsilon_{M+1}, \varepsilon_{M+2}, \ldots, \varepsilon_{K-1}, \varepsilon_{K}) \]

\[ d\varepsilon_K d\varepsilon_{K-1} \cdots d\varepsilon_{M+2} d\varepsilon_{M+1} d\varepsilon_1, \]

where \( J \) is the Jacobian whose elements are given by:

\[ J_{ih} = \frac{\partial[V_i - V_{i+1} + \varepsilon_i]}{\partial e_{h+1}^{*}} \quad ; \; i, h = 1, 2, \ldots, M - 1 \]

\[ P(e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0) = |J| \int_{\tilde{\varepsilon}_{M+1,1}=-\infty}^{V_{M+1}} \int_{\tilde{\varepsilon}_{M+2,1}=-\infty}^{V_{M+2}} \cdots \int_{\tilde{\varepsilon}_{K-1,1}=-\infty}^{V_{K-1}} \int_{\tilde{\varepsilon}_{K,1}=-\infty}^{V_{K}} g(V_1 - V_2, V_1 - V_3, \ldots, V_1 - V_M, \tilde{\varepsilon}_{M+1,1}, \tilde{\varepsilon}_{M+2,1}, \ldots, \tilde{\varepsilon}_{K,1}) \]

\[ d\tilde{\varepsilon}_{K,1} d\tilde{\varepsilon}_{K-1,1} \cdots d\tilde{\varepsilon}_{M+1,1} \]
Specific Model Structures

- The MDCEV model structure

\[
P\left( e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0 \right)
\]

\[
= |J| \int_{\varepsilon_1 = -\infty}^{\varepsilon_1 = +\infty} \left\{ \left( \prod_{i=2}^{M} \frac{1}{\sigma} \lambda \left[ \frac{V_1 - V_i + \varepsilon_1}{\sigma} \right] \right) \right\} \times \left\{ \prod_{s=M+1}^{K} \Lambda \left[ \frac{V_1 - V_s + \varepsilon_1}{\sigma} \right] \right\} \frac{1}{\sigma} \lambda \left( \frac{\varepsilon_1}{\sigma} \right) d\varepsilon_1
\]

\[
|J| = \left( \prod_{i=1}^{M} c_i \right) \left( \sum_{i=1}^{M} \frac{1}{c_i} \right), \text{ where } c_i = \left( \frac{1 - \alpha_i}{e_i^* + \gamma_i p_i} \right)
\]

\[
P\left( e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0 \right) = \frac{1}{\sigma^{M-1}} \left[ \prod_{i=1}^{M} c_i \right] \left[ \sum_{i=1}^{M} \frac{1}{c_i} \right] \left[ \prod_{k=1}^{K} \left( \sum_{i=1}^{M} e_i^{V_i/\sigma} \right)^M \right] (M - 1)!
\]

- Can be derived from the differenced form also
The MDCEV model structure

Probability of the consumption pattern of the goods (rather than the expenditure pattern) is

\[
P(x_1^*, x_2^*, x_3^*, \ldots, x_M^*, 0, 0, \ldots, 0) = \frac{1}{p_1} \cdot \frac{1}{\sigma^{M-1}} \left[ \prod_{i=1}^{M} f_i \right] \left[ \sum_{i=1}^{M} \frac{p_i}{f_i} \right] \left[ \prod_{i=1}^{M} e^{v_i / \sigma} \right] \left( \sum_{k=1}^{K} e^{v_k / \sigma} \right)^{M-1} (M-1)!,
\]

where

\[
f_i = \left( \frac{1 - \alpha_i}{x_i^* + \gamma_i} \right)
\]
The MDCGEV model

- Generalized Extreme Value error structure

- A nested logit example with four alternatives

- Start with the following expression for density function

\[
\Lambda(e_1 < s_1, e_2 < s_2, e_3 < s_3, e_4 < s_4) = \exp\left[-\left(\frac{e^{-s_1}}{\sigma_A} + \frac{e^{-s_2}}{\sigma_A}\right)^{\theta_A} - \left(\frac{e^{-s_3}}{\sigma_B} + \frac{e^{-s_4}}{\sigma_B}\right)^{\theta_B}\right]
\]
The MDCGEV model (nested logit example)

\[
P(x_1^*, 0, 0, 0) = \frac{A_{12}^{\theta_A - 1} \cdot e^{V_1 / \sigma_{\theta_A}}}{H}
\]

\[
P(x_1^*, x_2^*, 0, 0) = \frac{1}{\sigma} \left| J \right| \frac{e^{V_1 / \sigma_{\theta_A}} \cdot e^{V_2 / \sigma_{\theta_A}} \cdot A_{12}^{\theta_A - 2} \left( \frac{A_{12}^{\theta_A} H}{H} + \frac{1 - \theta_A}{\theta_A} \right)}{H}
\]

\[
P(x_1^*, 0, x_3^*, 0) = \frac{1}{\sigma} \left| J \right| \frac{e^{V_1 / \sigma_{\theta_A}} \cdot e^{V_3 / \sigma_{\theta_A}} \cdot A_{12}^{\theta_A - 1} B_{34}^{\theta_B - 1}}{H^2}
\]

\[
P(x_1^*, x_2^*, x_3^*, 0) = \frac{1}{\sigma^2} \left| J \right| \frac{e^{V_1 / \sigma_{\theta_A}} \cdot e^{V_2 / \sigma_{\theta_A}} \cdot e^{V_3 / \sigma_{\theta_A}} \cdot A_{12}^{\theta_A - 2} B_{34}^{\theta_B - 1} \left( \frac{2 A_{12}^{\theta_A} H}{H} + \frac{1 - \theta_A}{\theta_A} \right)}{H^2}
\]

\[
P(x_1^*, x_2^*, x_3^*, x_4^*) = \frac{1}{\sigma^3} \left| J \right| \frac{e^{V_1 / \sigma_{\theta_A}} \cdot e^{V_2 / \sigma_{\theta_A}} \cdot e^{V_3 / \sigma_{\theta_A}} \cdot e^{V_4 / \sigma_{\theta_B}} \cdot A_{12}^{\theta_A - 2} B_{34}^{\theta_B - 1}}{H^2}
\]

\[
\left\{ \frac{6 A_{12}^{\theta_A} B_{34}^{\theta_B}}{H^2} + \frac{2(1 - \theta_A)}{\theta_A} \cdot \frac{B_{34}^{\theta_B}}{H} + \frac{2(1 - \theta_B)}{\theta_B} \cdot A_{12}^{\theta_A} \cdot \frac{(1 - \theta_A)}{\theta_A} \cdot \frac{(1 - \theta_B)}{\theta_B} \right\}
\]

\[
A_{12} = \left( e^{V_1 / \sigma_{\theta_A}} + e^{V_2 / \sigma_{\theta_A}} \right)
\]

\[
B_{34} = \left( e^{V_3 / \sigma_{\theta_B}} + e^{V_4 / \sigma_{\theta_B}} \right)
\]

\[
H = A_{12}^{\theta_A} + B_{34}^{\theta_B}
\]
The Mixed MDCEV model

\[ P(e_1^*, e_2^*, e_3^*, \ldots, e_M^*, 0, 0, \ldots, 0) \]

\[ = \int_{\eta} \frac{1}{\sigma^{M-1}} \left[ \prod_{i=1}^{M} c_i \right] \left[ \sum_{i=1}^{M} \frac{1}{c_i} \right] \left[ \frac{\prod_{i=1}^{M} e^{(V_i + \eta_i) / \sigma}}{\left( \sum_{k=1}^{K} e^{(V_k + \eta_k) / \sigma} \right)^M} \right] \]

\[ (M - 1)! dF(\eta). \]
The Mixed MDCEV model - Heteroscedastic structure

The heteroscedastic structure may be specified in the form of the following covariance matrix for \( \varepsilon = (\varepsilon_{k1}, \varepsilon_{k2}, \varepsilon_{k3}, \varepsilon_{k4}) \):

\[
\text{Cov}(\varepsilon) = \frac{\pi^2 \sigma^2}{6} \begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix} + \begin{bmatrix}
\omega_1^2 & 0 & 0 & 0 \\
0 & \omega_2^2 & 0 & 0 \\
0 & 0 & \omega_3^2 & 0 \\
0 & 0 & 0 & \omega_4^2
\end{bmatrix}
\]

Two ways to proceed with a normalization

- Normalize \( \sigma \) and estimate the heteroscedastic covariance matrix of \( \eta \text{ (i.e., } \omega_1, \omega_2, \omega_3 \text{ and } \omega_4) \)
- Normalize one of the \( \omega_k \) terms instead of the \( \sigma \) term
The Mixed MDCEV model - General error covariance structure

- Allows correlation in unobserved factors influencing the baseline utility of alternatives
- Requires appropriate identification normalizations to be placed on $\sigma$ and the covariance matrix of $\eta$
- One way to achieve identification in the most general error covariance structure, and when there is price variation:
  - Normalize the scale parameter $\sigma$ to be a small value such that the variance of the minimum variance alternative exceeds $\pi^2 \sigma^2 / 6$
  - Normalize $\omega_k$ for the minimum variance alternative $k$ to zero
  - Normalize all correlations of this minimum variance alternative with other alternatives to zero
  - These normalizations leave only $K(K-1)/2$ parameters to be estimated, and are adequate for identification
Empirical Analysis

- Increasing dependence on automobiles
- Wide-ranging impacts of automobile dependency
  - Household level
  - Community level
  - Regional level
A widely used indicator of automobile dependency is vehicle holdings and use

- 92% of US households owned at least one motorized vehicle in 2003 (compared to 80% in the early 1970s)
- Household VMT has increased 300% between 1997-2001 relative to a population increase of 30% during same period

Important to examine vehicle holdings and usage

- Travel demand forecasting
- Transportation policy analysis
Examine several dimensions of household vehicle holdings and usage decisions

- Number of vehicles owned
- Vehicle body type
- Vehicle age (i.e., vintage)
- Vehicle make and model
- Vehicle usage
- Incorporate a comprehensive set of determinants of vehicle holdings and usage decisions
  - Household demographics
  - Individual characteristics
  - Vehicle characteristics
  - Built environment characteristics
- Develop a comprehensive econometric model to analyze the many dimensions of vehicle holdings and use that accommodates for
  - Multiple discreteness
  - Satiation effects
Data

- 2000 San Francisco Bay Area travel survey (BATS)
  - Designed and administered by MORPACE International Inc.
  - 2-day survey of 15000 households
  - Information on vehicle fleet mix of households, individual and household socio-demographics, individual characteristics and activity episodes

- Data on vehicle make/model attributes from secondary data sources
  - Consumer Guides
  - EPA Fuel Economy Guide

- Land use/Demographic coverage data from MTC of San Francisco Bay area

- GIS layer of bicycle facilities from MTC of San Francisco Bay area

- Census 2000 Tiger files
Sample Characteristics

- Final sample: 8107 households
- 10 motorized vehicle types
  - Coupe
  - Mini/Subcompact Sedan
  - Compact Sedan
  - Mid-size Sedan
  - Large Sedan
  - Hatchback/Station Wagon
  - Sports Utility Vehicle (SUV)
  - Pickup Truck
  - Minivan
  - Van
- 2 vintages considered for each motorized vehicle type
  - New vehicles (age of the vehicle less than or equal than 5 years)
  - Old Vehicles (age of the vehicle is more than 5 years)
- Twenty-one vehicle types/vintages studied including
  - 20 motorized vehicle type/vintages
  - Non-motorized form of transportation
Classification of Vehicle type/vintage

- Coupe Old: 33 makes/models
- Coupe New: 23 makes/models
- Sedan Mini/Subcompact Old: 10 makes/models
- Sedan Mini/Subcompact New: 7 makes/models
- Sedan Compact Old: 25 makes/models
- Sedan Compact New: 19 makes/models
- Sedan Mid-size Old: 24 makes/models
- Sedan Mid-size New: 21 makes/models
- Sedan Large Old: 16 makes/models
- Sedan Large New: 12 makes/models
- Hatchback/Station Wagon Old: 23 makes/models
- Hatchback/Station Wagon New: 12 makes/models
- SUV Old: 15 makes/models
- SUV New: 23 makes/models
- Pickup Truck Old: 12 makes/models
- Pickup Truck New: 13 makes/models
- Minivan Old: 13 makes/models
- Minivan New: 15 makes/models
- Van Old: 6 makes/models
- Van New: 5 makes/models
- Non-motorized vehicles
 Distribution of Vehicles

<table>
<thead>
<tr>
<th>Number of vehicles owned by the household</th>
<th>Total No. of households</th>
<th>% of households</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4459</td>
<td>55%</td>
</tr>
<tr>
<td>2</td>
<td>2918</td>
<td>36%</td>
</tr>
<tr>
<td>3</td>
<td>644</td>
<td>8%</td>
</tr>
<tr>
<td>4 or more</td>
<td>86</td>
<td>1%</td>
</tr>
</tbody>
</table>
### Descriptive Statistics of Vehicle Type/Vintage Holdings

<table>
<thead>
<tr>
<th>Vehicle type/ vintage</th>
<th>Total number (%) of households owning</th>
<th>Annual Mileage</th>
<th>No. of households who own (%)</th>
<th>Only Vehicle type/ vintage</th>
<th>Vehicle type/ vintage and other Vehicle type/ vintages</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Coupe</td>
<td>389 (5%)</td>
<td>7763</td>
<td>132 (34%)</td>
<td>257 (66%)</td>
<td></td>
</tr>
<tr>
<td>Old Coupe</td>
<td>1024 (13%)</td>
<td>7766</td>
<td>374 (37%)</td>
<td>650 (63%)</td>
<td></td>
</tr>
<tr>
<td>New Subcompact Sedan</td>
<td>292 (4%)</td>
<td>7838</td>
<td>127 (43%)</td>
<td>165 (57%)</td>
<td></td>
</tr>
<tr>
<td>Old Subcompact Sedan</td>
<td>513 (6%)</td>
<td>9570</td>
<td>238 (46%)</td>
<td>275 (54%)</td>
<td></td>
</tr>
<tr>
<td>New Compact Sedan</td>
<td>767 (9%)</td>
<td>8321</td>
<td>342 (45%)</td>
<td>425 (55%)</td>
<td></td>
</tr>
<tr>
<td>Old Compact Sedan</td>
<td>1175 (14%)</td>
<td>9614</td>
<td>495 (42%)</td>
<td>680 (58%)</td>
<td></td>
</tr>
<tr>
<td>New Midsize Sedan</td>
<td>987 (12%)</td>
<td>7688</td>
<td>361 (37%)</td>
<td>626 (63%)</td>
<td></td>
</tr>
<tr>
<td>Old Midsize Sedan</td>
<td>1543 (19%)</td>
<td>9342</td>
<td>636 (41%)</td>
<td>907 (59%)</td>
<td></td>
</tr>
<tr>
<td>New Large Sedan</td>
<td>250 (3%)</td>
<td>7418</td>
<td>71 (28%)</td>
<td>179 (72%)</td>
<td></td>
</tr>
<tr>
<td>Old Large Sedan</td>
<td>377 (5%)</td>
<td>8339</td>
<td>151 (40%)</td>
<td>226 (60%)</td>
<td></td>
</tr>
<tr>
<td>New Station Wagon</td>
<td>242 (3%)</td>
<td>7869</td>
<td>80 (33%)</td>
<td>162 (67%)</td>
<td></td>
</tr>
<tr>
<td>Old Station Wagon</td>
<td>728 (9%)</td>
<td>8248</td>
<td>254 (35%)</td>
<td>474 (65%)</td>
<td></td>
</tr>
<tr>
<td>New SUV</td>
<td>707 (9%)</td>
<td>8920</td>
<td>245 (35%)</td>
<td>462 (65%)</td>
<td></td>
</tr>
<tr>
<td>Old SUV</td>
<td>711 (9%)</td>
<td>9813</td>
<td>213 (30%)</td>
<td>498 (70%)</td>
<td></td>
</tr>
<tr>
<td>New Pickup Truck</td>
<td>578 (7%)</td>
<td>8887</td>
<td>153 (26%)</td>
<td>425 (74%)</td>
<td></td>
</tr>
<tr>
<td>Old Pickup Truck</td>
<td>1198 (15%)</td>
<td>8679</td>
<td>301 (25%)</td>
<td>897 (75%)</td>
<td></td>
</tr>
<tr>
<td>New Minivan</td>
<td>459 (6%)</td>
<td>9156</td>
<td>115 (25%)</td>
<td>344 (75%)</td>
<td></td>
</tr>
<tr>
<td>Old Minivan</td>
<td>480 (6%)</td>
<td>9890</td>
<td>130 (27%)</td>
<td>350 (73%)</td>
<td></td>
</tr>
<tr>
<td>New Van</td>
<td>39 (1%)</td>
<td>10640</td>
<td>8 (21%)</td>
<td>31 (79%)</td>
<td></td>
</tr>
<tr>
<td>Old Van</td>
<td>122 (2%)</td>
<td>8203</td>
<td>33 (27%)</td>
<td>89 (73%)</td>
<td></td>
</tr>
<tr>
<td>Non-Motorized form of transportation</td>
<td>201 (3%)</td>
<td>2695</td>
<td>-</td>
<td>201 (100%)</td>
<td></td>
</tr>
</tbody>
</table>
Empirical Results

- **Variables considered**
  - **Household socio-demographics**
    - Household income, presence of children in the household, presence of a senior adult in the household, household size and number of employed people in the household
  - **Household location attributes**
    - Area type variables (central business district, urban zone, suburban zone and rural zone), residential density and employment density variables
  - **Built environment characteristics of the residential neighborhood**
    - Percentages and absolute values of acreage in residential, commercial/industrial, and other land-use categories; fractions and number of single family and multi-family dwelling units, and fractions and number of households living in single family and multi-family dwelling units, bikeway density, street block density, highway density
  - **Characteristics of the household head**
    - Age (classified into less than 30 years of age, 31 to 45 years of age and greater than 45 years of age), gender and ethnicity (primarily, Caucasian, African-American, Hispanic, Asian and Other)
  - **Vehicle Characteristics**
    - Purchase price, fuel cost, seating capacity, luggage volume, engine size, number of cylinders, front headroom space, front legroom space, rear headroom space, rear legroom space, standard payload capacity (for pickup trucks only), wheelbase, length, height, width, horse power, vehicle weight, type of fuel used, amount of greenhouse gas emissions (tons/year), types of drive wheels, type of vehicle make
MDCEV model – Effects of Household Demographics

- Medium income (35-90K) and high income (>90K) households have a high baseline preference for new SUVs as compared to low-income households and a low preference for old vans.
- High income households have a lower baseline preference for old vehicles compared to low/middle income households.
- High income households are less likely to undertake activities using non-motorized forms of transportation.
- Households with very small children (less than 4 years of age) are more likely to use compact sedans, mid-size sedans, and SUVs than other households.
- Households with kids between 5 and 15 years of age have a high baseline preference for minivans than other households.
- Households with senior adults (greater than 65 years) are more likely to use compact, mid-size, and large sedans relative to coupes and subcompact sedans.
- As the size of the household increases, the household is more likely to use mid-size sedans, large sedans, station wagons, SUVs, pickup trucks, minivans and vans.
- Household with more number of employed members have a high baseline preference for new vehicle types such as subcompact sedans and compact sedans while a low baseline preference for large sedans and minivans.
MDCEV model – Effects of Household Location Characteristics

- Households residing in the suburban zones are less likely to own and use old vehicles relative to households in urban zones.
- Households residing in the suburban and rural zones are more likely to own and use pickup trucks relative to urban households.

MDCEV model – Effects of Built Environment Characteristics of the Residential Neighborhood

- Households located in highly residential/commercial areas are less likely to prefer large vehicle types such as pickup trucks and vans, irrespective of the age of the vehicle.
- Households located in a neighborhood with high bike lane density have a high baseline preference for non-motorized modes of transportation.
- Households located in a neighborhood with high street block density are more likely to prefer smaller vehicle types (such as subcompact and compact sedans), and older vehicles, relative to new vehicles.
MDCEV model – Characteristics of the Household Head

- Older households (i.e., households whose heads are greater than 30 years) are generally more likely to own vehicles of an older vintage compared to younger households (i.e., households whose heads are less than or equal to 30 years of age).
- Older households are more likely to own minivans and old vans, and travel by non-motorized forms of transportation.
- Households have higher baseline preference for older and larger vehicles if the male is the oldest member (or only adult) in the household relative to households with the female being the oldest member (or only adult).
- Asians more likely to own sedans and new minivans, and less likely to own pickup trucks, than other races.

MDCEV model – Random Error Components/Coefficients

- Households preferring old coupes due to unobserved factors also prefer new coupes.
- Intangible unobserved factors that affect utilities of all old vehicles.
## MNL model for Vehicle Make/Model Choice

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase Price (in $)/Income (in $/yr) [x 10]</td>
<td>-0.173</td>
<td>-5.71</td>
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<tr>
<td>Mean Effect</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>-0.064</td>
<td>-4.44</td>
</tr>
<tr>
<td>Fuel Cost (in $/yr) /Income (in $/yr) [x 10]</td>
<td>-0.003</td>
<td>-1.61</td>
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<tr>
<td>Seat Capacity * Household Size less than equal to 2 dummy variable</td>
<td>-0.075</td>
<td>-5.11</td>
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<tr>
<td>Luggage Volume (in 10s of cubic feet)</td>
<td>0.023</td>
<td>3.54</td>
</tr>
<tr>
<td>Standard Payload Capacity (for Pickup Trucks only) (in 1000 lbs)</td>
<td>0.196</td>
<td>5.13</td>
</tr>
<tr>
<td>Horsepower (in HP) /Vehicle Weight (in lbs) [in 10s]</td>
<td>1.102</td>
<td>4.89</td>
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<tr>
<td>Engine Size (in liters)</td>
<td>-0.045</td>
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<tr>
<td>Dummy variable for All-Wheel-Drive (base: rear-wheel-drive)</td>
<td>-0.214</td>
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<td>Dummy Variable for Vehicle Make - Chevy</td>
<td>-0.149</td>
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<td>Dummy Variable for Vehicle Make - Ford</td>
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<td>Dummy Variable for Vehicle Make - Honda</td>
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<td>Dummy Variable for Vehicle Make - Toyota</td>
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<td>Dummy Variable for Vehicle Make - Cadillac</td>
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<td>Dummy Variable for Vehicle Make - Volkswagen</td>
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<tr>
<td>Dummy Variable for Vehicle Make - Dodge</td>
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<tr>
<td>Amount of Greenhouse Gas Emissions (in 10s of tons/yr)</td>
<td>-0.429</td>
<td>-2.71</td>
</tr>
<tr>
<td>Dummy variable for Premium Fuel (base: regular fuel)</td>
<td>-0.552</td>
<td>-5.01</td>
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</tbody>
</table>
Satiation Effects

- All the satiation parameters are very significantly different from 1
- Middle and High income households are more likely to get satiated with the increasing use of any vehicle type/vintage compared to low income households
- Low income households are least likely to get satiated with the increasing use of old subcompact sedans, new and old compact sedans, and old midsize sedans
- Satiation effect is highest for non-motorized mode of transportation compared to all vehicle type/vintage categories

Logsum Parameters

- Indicate the presence of common unobserved attributes that affect the utilities of all makes/models corresponding to old SUV, old minivan, new minivan, old van, and new van vehicle type/vintage categories
## Application of the Model

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Impact of a 25% increase in bike lane density</th>
<th>Impact of a 25% increase in street block density</th>
<th>Impact of a 25% increase in fuel cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% change in holdings of vehicle type</td>
<td>% change in overall use of vehicle type</td>
<td>% change in holdings of vehicle type</td>
</tr>
<tr>
<td>Compact Car</td>
<td>-2.2%</td>
<td>8.5%</td>
<td>3.4%</td>
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<tr>
<td>Midsize and Large Sedan</td>
<td>-2.1%</td>
<td>-</td>
<td>-0.8%</td>
</tr>
<tr>
<td>SUV</td>
<td>-0.4%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pickup Truck</td>
<td>-1.4%</td>
<td>-2.1%</td>
<td>-1.7%</td>
</tr>
<tr>
<td>Minivan and Van</td>
<td>-0.7%</td>
<td>-</td>
<td>-0.6%</td>
</tr>
<tr>
<td>Non-motorized modes of transport</td>
<td>7.4%</td>
<td>-4.0%</td>
<td>-3.3%</td>
</tr>
</tbody>
</table>
Directions for further research

- Accommodating more than one constraint in the utility maximization problem (for example, recognizing both time and money constraints in activity type choice and duration models)
- Incorporating latent consideration sets in a theoretically appropriate way within the MDCEV structure
- Using more flexible utility structures that can handle both complementarity as well as substitution among goods, and that do not impose the constraints of additive separability
Model with an Outside Good

\[ U(x) = \frac{1}{\alpha_1} \exp(\varepsilon_1)x_1^{\alpha_1} + \sum_{k=2}^{K} \frac{1}{\alpha_k} \exp(\beta'z_k + \varepsilon_k) \left\{ (x_k + 1)^{\alpha_k} - 1 \right\} \]

\[ U(x) = \frac{1}{\alpha_1} \exp(\varepsilon_1)x_1^{\alpha_1} + \sum_{k=2}^{K} \gamma_k \exp(\beta'z_k + \varepsilon_k) \ln \left( \frac{x_k}{\gamma_k} + 1 \right) \]

\[ U(x) = \frac{1}{\alpha} \exp(\varepsilon_1)x_1^{\alpha} + \sum_{k=2}^{K} \frac{\gamma_k}{\alpha} \exp(\beta'z_k + \varepsilon_k) \left\{ \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha} - 1 \right\} \]
Comparison With Earlier Multiple Discrete-Continuous Models

- Kim et al.'s model

\[ U = \sum_{k=1}^{K} \psi_k^1 (x_k + \gamma_k^1)\alpha_k^1 \]

\[ U = \sum_{k=1}^{K} \psi_k^1 \left\{ (x_k + \gamma_k^1)^{\alpha_k^1} - (\gamma_k^1)^{\alpha_k^1} \right\} \]

\[ \tau_k = \tau_k^1 + \ln \left( \frac{\alpha_k^1}{\alpha_1^1} \right) \quad \forall \ k \]

\[ \alpha_k = \alpha_k^1 \quad \forall \ k \]

\[ \beta = \beta^1 \]
Models in environmental economics

\[ U = \sum_{k=1}^{K} \psi_k^2 \ln(x_k + \gamma_k^2) \]

\[ U = \sum_{k=1}^{K} \psi_k^2 \ln\left(\frac{x_k}{\gamma_k^2} + 1\right) \]

\[ \tau_k = \tau_k^2 + \ln\left(\frac{\gamma_1^2}{\gamma_k^2}\right) \quad \forall \ k \]

\[ \gamma_k = \gamma_k^2 \quad \forall \ k \]

\[ \beta = \beta^2 \]
\[ P\left( x_1^*, x_2^*, x_3^*, \ldots, x_M^*, 0, 0, \ldots, 0 \right) \]
\[ = |J|_x \int_{\epsilon_1 = -\infty}^{\epsilon_1 = +\infty} \left\{ \left( \prod_{i=2}^{M} \frac{1}{\sigma} \lambda \left[ \frac{V_1-V_i + \epsilon_1}{\sigma} \right] \right) \times \left( \prod_{s=M+1}^{K} \Lambda \left[ \frac{V_1-V_s + \epsilon_1}{\sigma} \right] \right) \right\} \frac{1}{\sigma} \lambda \left( \frac{\epsilon_1}{\sigma} \right) d\epsilon_1 \]
\[ = |J|_x \left\{ \left( \prod_{i=2}^{M} \frac{1}{\sigma} \lambda \left[ \frac{V_1-V_i}{\sigma} \right] \right) \times \left( \prod_{s=M+1}^{K} \Lambda \left[ \frac{V_1-V_s}{\sigma} \right] \right) \right\}, \quad \text{for } \epsilon_1 = 0 \]
\[ = |J|_x \left\{ \prod_{i=2}^{M} \frac{\exp(-g_i/\sigma)}{\sigma} \right\} \left\{ \exp\left( -\sum_{k=2}^{K} \exp(-g_k/\sigma) \right) \right\}, \quad \text{for } \epsilon_1 = 0 \]

where, \( |J|_x = \left( \prod_{i=1}^{M} f_i \right) \left( \sum_{i=1}^{M} \frac{p_i}{f_i} \right), \) \( f_i = \left( \frac{1-\alpha_i}{x_i^* + \gamma_i} \right), \) and \( g_k = V_1-V_k \)
\[
P\left( x^*_1, x^*_2, x^*_3, \ldots, x^*_M, 0, 0, \ldots, 0 \right) \\
= |J|_x \left\{ \prod_{i=2}^{M} \frac{\exp(-g_i / \sigma)}{\sigma} \right\} \left\{ \exp\left( -\sum_{k=2}^{K} \exp(-g_k / \sigma) \right) \right\}, \text{ for } \varepsilon_i = 0; \ l = 1 \\
= \frac{1}{p_l} |J|_x \left\{ \exp(+g_l / \sigma) \right\}^{M-1} \left\{ \prod_{i=1}^{M} \frac{\exp(-g_i / \sigma)}{\sigma} \right\} \left\{ \exp\left( -\sum_{k=1}^{K} \exp(-g_k / \sigma) \right) \right\} \exp(+g_l / \sigma), \text{ for } \varepsilon_i = 0; \ l \leq M \\
= |J|_x \left\{ \prod_{i=2}^{M} \frac{\exp(-g_i / \sigma)}{\sigma} \right\} \left\{ \sum_{k=1}^{K} \exp(-g_k / \sigma) \right\}^{-M} \\
\left\{ (M-1)! \exp(d_i)(-1)^{M+1} \left( d_i^{M-1} - (M-1)d_i^{M-2} + (M-1)(M-2)d_i^{M-3} + \ldots + (-1)^{M-1}(M-1)! \right) \right\}, \text{ for } \varepsilon_i = 0; \ l > M \\
\text{where, } |J|_x = \left( \prod_{i=1}^{M} f_i \right) \left( \sum_{i=1}^{M} p_i f_i \right), \ f_i = \left( \frac{1-\alpha_i}{x_i + \gamma_i} \right), \ g_k = V_1 - V_k, \text{ and } d_i = -\left\{ \sum_{k=1}^{K} \exp(-g_k / \sigma) \right\} \exp(+g_l / \sigma)
Three Good Case: Probability of Choice of Only Outside Good

\[ P^1(x^*_1, 0, 0) = \exp\left[ -\sum_{k=2}^{3} \exp(-g_k / \sigma) \right] \]

\[ P^2(x^*_1, 0, 0) = \left\{ 1 + \exp(-g_3 / \sigma) \right\}^{-1} \left\{ 1 - \exp\left[ -\left\{ 1 + \exp(-g_3 / \sigma) \right\} \{ \exp(+g_2 / \sigma) \} \right]\right\} \]

\[ P^3(x^*_1, 0, 0) = \frac{\exp(V_1 / \sigma)}{\sum_{k=1}^{3} \exp(V_k / \sigma)} = \frac{1}{1 + \exp(-g_2 / \sigma) + \exp(-g_3 / \sigma)} \]
Utility of outside good is assumed to be deterministic (i.e., $\epsilon_1 = 0$)

A three-good example
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Estimate t-stat</td>
<td>Estimate t-stat</td>
<td>Estimate t-stat</td>
<td>Estimate t-stat</td>
<td>Estimate t-stat</td>
<td>Estimate t-stat</td>
</tr>
<tr>
<td>Baseline marginal utility ($\beta_k$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Passenger car</td>
<td>-2.204 -38.81</td>
<td>-4.409 -38.81</td>
<td>-5.800 -29.74</td>
<td>-10.817 -12.05</td>
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<tr>
<td>Pickup-Truck</td>
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<td>-5.497 -42.08</td>
<td>-7.230 -27.57</td>
<td>-13.484 -12.20</td>
<td>-2.549 -41.17</td>
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<tr>
<td>Van</td>
<td>-</td>
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<td></td>
<td></td>
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<tr>
<td>Satiation Parameters</td>
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</tr>
<tr>
<td>$\alpha_{\text{Passenger Car}}$</td>
<td>0.619 38.28</td>
<td>0.239 7.40</td>
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<td>0.000 (fixed)</td>
<td>-1.000 (fixed)</td>
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<tr>
<td>$\alpha_{\text{SUV}}$</td>
<td>0.886 60.16</td>
<td>0.773 26.24</td>
<td>0.701 16.19</td>
<td>0.443 5.10</td>
<td>0.000 (fixed)</td>
<td>-1.000 (fixed)</td>
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<td>$\alpha_{\text{Pick-up}}$</td>
<td>0.796 51.36</td>
<td>0.592 19.10</td>
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<td>0.000 (fixed)</td>
<td>-1.000 (fixed)</td>
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<tr>
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<td>0.762 20.40</td>
<td>0.687 12.93</td>
<td>0.416 4.00</td>
<td>0.000 (fixed)</td>
<td>-1.000 (fixed)</td>
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<tr>
<td>$\alpha_{\text{Van}}$</td>
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<td>0.500 3.88</td>
<td>0.069 0.28</td>
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<td>$\gamma_{\text{Passenger Car}}$</td>
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<td>1.000 (fixed)</td>
<td>1.000 (fixed)</td>
<td>1.000 (fixed)</td>
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<td>11.469 14.79</td>
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<td>1.000 (fixed)</td>
<td>1.000 (fixed)</td>
<td>1.000 (fixed)</td>
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<td>1.000 (fixed)</td>
<td>1.000 (fixed)</td>
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<td>1.000 (fixed)</td>
<td>1.000 (fixed)</td>
<td>1.000 (fixed)</td>
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<td>1.000 (fixed)</td>
<td>1.000 (fixed)</td>
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<td>-9648.48</td>
<td>-9648.48</td>
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<td>-9218.89</td>
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Specifications for the “No Outside Good” case with price variables

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model 1 (Consumption-based)</th>
<th>Model 2 (Consumption-based)</th>
<th>Model 3 (Expenditure-based)</th>
<th>Model 4 (Expenditure-based)</th>
<th>Model 5 (Expenditure-based)</th>
<th>Model 6 (Expenditure-based)</th>
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</thead>
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<tr>
<td></td>
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<td>t-stat</td>
<td>Estimate</td>
<td>t-stat</td>
<td>Estimate</td>
<td>t-stat</td>
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<tr>
<td>Baseline marginal utility ($\beta_k$)</td>
<td>-5.088</td>
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<td>-5.087</td>
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<td>-5.086</td>
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<td>Satiation Parameters</td>
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<tr>
<td>$\alpha_{Passenger Car}$</td>
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<td>0.00</td>
<td>0.000</td>
<td>0.00</td>
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<td>(fixed)</td>
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<td>(fixed)</td>
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<td>$\gamma_{Pick-up}$</td>
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<td>-9348.95</td>
<td>-8803.51</td>
<td>-8939.42</td>
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Specifications for case with outside good and with price variables

<table>
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<tr>
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<th>Model 2 (Consumption-based)</th>
<th>Model 3 (Consumption-based)</th>
<th>Model 4 (Consumption-based)</th>
<th>Model 5 (Consumption-based)</th>
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</thead>
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Specifications for case with outside good and with price variables (continued)

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<tr>
<th>Parameters</th>
<th>Model 1 (Expenditure-based)</th>
<th>Model 2 (Consumption-based)</th>
<th>Model 3 (Consumption-based)</th>
<th>Model 4 (Consumption-based)</th>
<th>Model 5 (Consumption-based)</th>
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<td>Est.</td>
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Conclusions

- Several issues associated with extant KT multiple discrete-continuous models were examined
  - A new utility function form was proposed that enables clarity in the role of each parameter in the utility specification
  - Identification considerations associated with the utility specification was presented and the MDCEV model was extended to the case of price variation across goods and to general error covariance structures
  - The relationship between earlier KT-based multiple discrete-continuous models was discussed
  - Several technical nuances and identification considerations of the multiple discrete-continuous model structure was illustrated through empirical examples
General econometric model structure and identification

\[ \tilde{U} = \sum_k \frac{\gamma_k}{\alpha_k^* \sigma} \left[ \exp\{\sigma \times (\beta'z_k + \epsilon_k)\} \right] \cdot \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k^*} - 1 \]

KT conditions for optimal expenditure for this modified utility function can be shown to be:

\[ V_k^* + \sigma \epsilon_k = V_1^* + \sigma \epsilon_1 \quad \text{if} \quad \epsilon_k^* > 0 \quad (k = 2, 3, \ldots, K) \]

\[ V_k^* + \sigma \epsilon_k < V_1^* + \sigma \epsilon_1 \quad \text{if} \quad \epsilon_k^* = 0 \quad (k = 2, 3, \ldots, K), \quad \text{where} \]

\[ V_k^* = \sigma \beta' z_k + (\alpha_k^* - 1) \ln \left( \frac{e_k^*}{\gamma_k p_k} + 1 \right) - \ln p_k \quad (k = 1, 2, 3, \ldots, K) \]

\[ = \sigma \beta' z_k + \sigma (\alpha_k - 1) \ln \left( \frac{e_k^*}{\gamma_k p_k} + 1 \right) - \ln p_k \quad (k = 1, 2, 3, \ldots, K) \]
Research objectives

- Reformulate the utility specification used in earlier studies
- Present identification considerations related to both the functional form as well as the stochastic nature of the utility specification
- Derive the MDCEV model expression for the case when there is price variation across goods and extend the MDCEV model to accommodate generalized extreme value (GEV)-based and other correlation structures
- Discuss the relationship between the models of Kim et al. (2002), the KT formulations used in Environmental Economics, and the MDCEV formulation
- Illustrate the technical issues related to the properties and identification of the MDCEV model through empirical illustrations